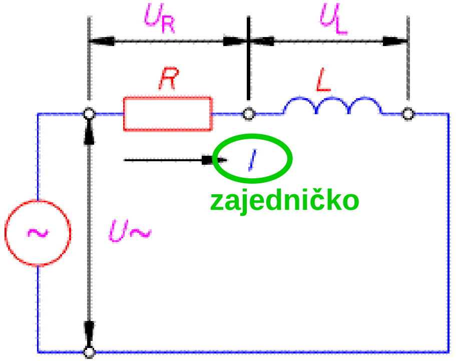
SPOJEVI IMPEDANCIJA I NJEZINIH KOMPONENATA

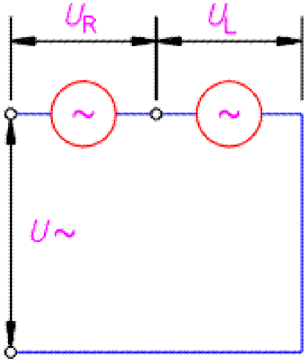
Induktivna impedancija

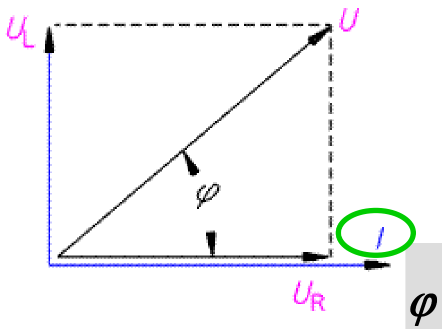
$U_R = I \cdot R = U \cdot \cos\varphi$

$U_L = I \cdot \omega L = U \cdot \sin\varphi$

$\underline{U} = U_R + U_L = I \cdot (R + j\omega L) = U \cdot e^{+j\varphi}$



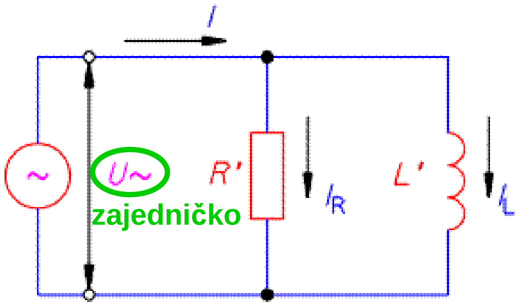


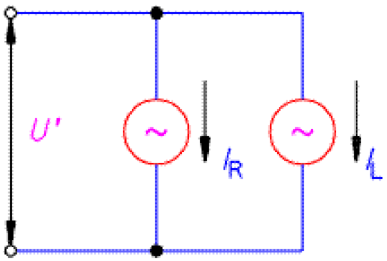


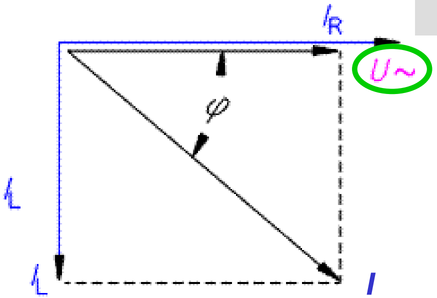
$U = I \cdot \sqrt{R^2 + (\omega L)^2}$

$Z = \sqrt{R^2 + (\omega L)^2}$

$\varphi = \text{arc tg}\,\frac{U_L}{U_R} = \text{arc tg}\,\frac{\omega L}{R}$







$\underline{Z} = R + j\omega L$

$I_R = \frac{U}{R'}$

$I_L = \frac{U}{\omega L'}$

$\underline{I} = I_R + jI_L = U\left(\frac{1}{R'} + j\frac{1}{\omega L'}\right) = I \cdot e^{+j\varphi}$

$Z = \frac{R' \cdot \omega L'}{\sqrt{R'^2 + \omega^2 L'^2}}$

$\varphi = \text{arc tg}\,\frac{\omega L'}{R'}$

$\underline{Z} = \frac{R' \cdot \omega^2 L'^2}{R'^2 + \omega^2 L'^2} + j\frac{R' \cdot \omega L'}{R'^2 + \omega^2 L'^2}$

ekvivalentne vrijednosti

$R = \frac{R' \cdot \omega^2 L'^2}{R'^2 + \omega^2 L'^2}$

$L = \frac{R' \cdot \omega L'}{R'^2 + \omega^2 L'^2}$

Kapacitivna impedancija

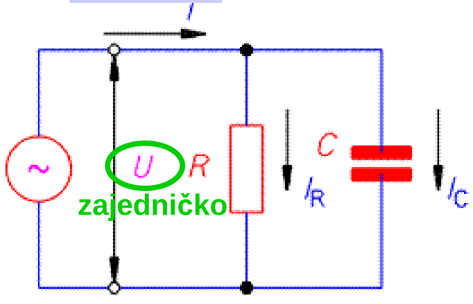
$$I_R = \frac{U}{R}$$

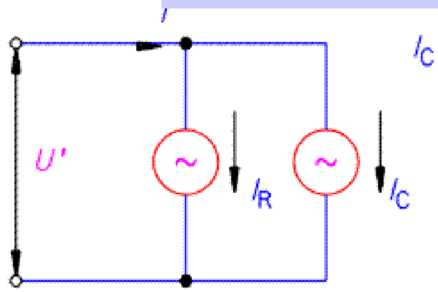
$$I_C = U \cdot \omega \cdot C$$

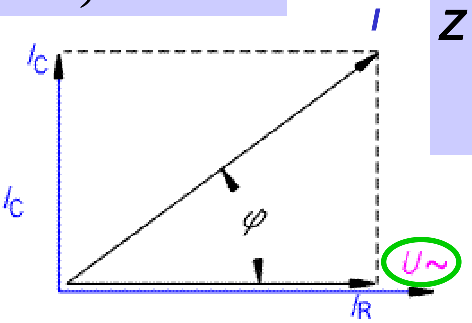
$$\underline{I} = U\left(\frac{1}{R} - j\omega C\right) = I \cdot e^{-j\varphi}$$

$$I = U\sqrt{\frac{1}{R^2} + \omega^2 C^2}$$

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \omega^2 C^2}}$$

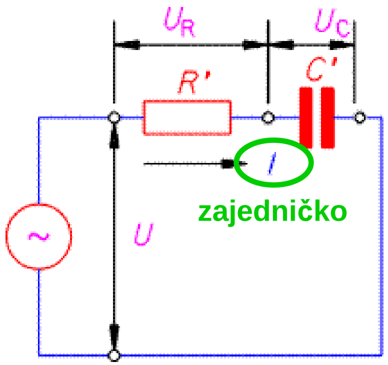


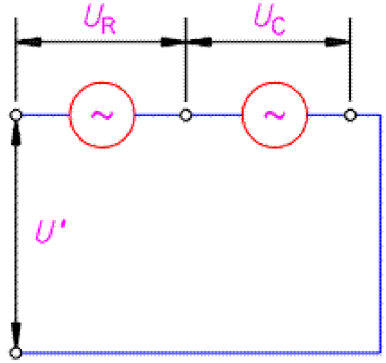


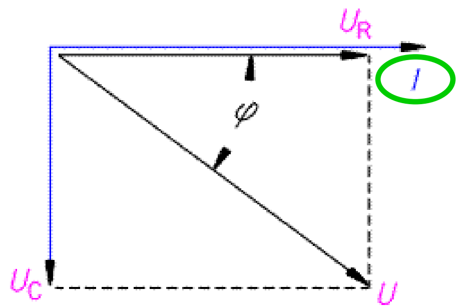


$$\varphi = \text{arc tg}\,\omega C R$$

$$\underline{Z} = \frac{1}{\frac{1}{R} - j\omega C}$$







$$U_R = I \cdot R'$$

$$U_C = I \cdot \frac{1}{\omega C'}$$

$$\underline{U} = I\left(R' - j\frac{1}{\omega C'}\right) = U \cdot e^{-j\varphi}$$

$$Z = \sqrt{R'^2 + \frac{1}{\omega^2 C'^2}}$$

$$\varphi = \text{arc tg}\frac{1}{R' \cdot \omega C'}$$

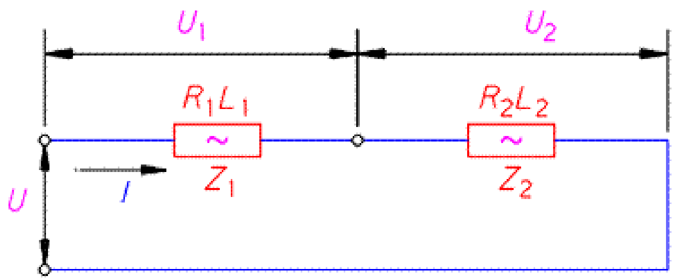
$$\underline{Z} = R' - \frac{1}{j\omega C'}$$

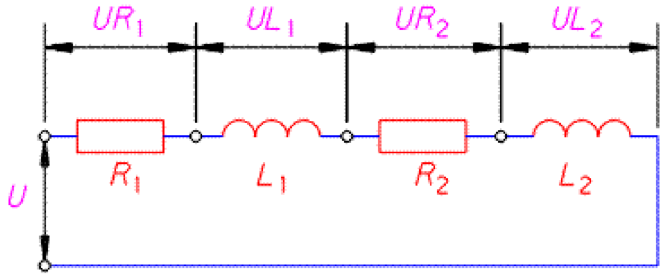
ekvivalentne vrijednosti

$$R' = \frac{R}{1 + \omega^2 C^2 R^2}$$

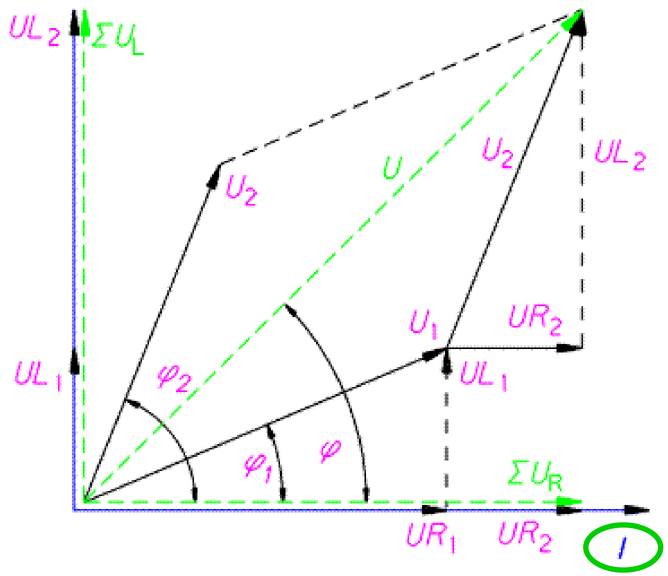
$$C' = \frac{1 + \omega^2 C^2 R^2}{\omega C R^2}$$

Serijski spoj impedancija





zajednička je struja



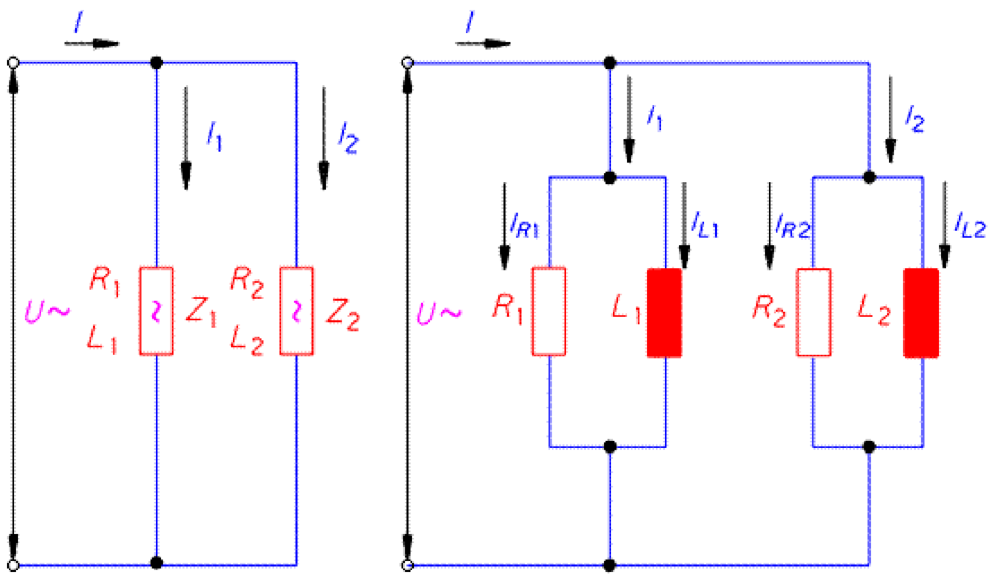
prema II Kirchhoffovom zakonu $\underline{U} = \sum U_R + j\sum U_L = U \cdot e^{+j\varphi}$

$$\underline{U} = I\left[(R_1 + R_2) + j(\omega L_1 + \omega L_2)\right]$$

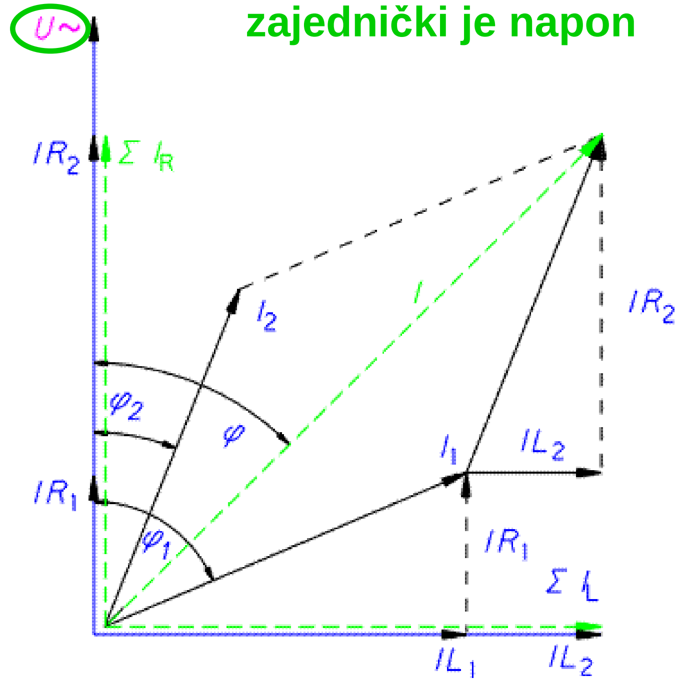
$$\underline{Z} = (R_1 + R_2) + j(\omega L_1 + \omega L_2) = Z \cdot e^{+j\varphi}$$

$$Z = \sqrt{(R_1 + R_2)^2 + (\omega L_1 + \omega L_2)^2} \qquad \varphi = \text{arc tg}\frac{\omega L_1 + \omega L_2}{R_1 + R_2}$$

Paralelni spoj impedancija



zajednički je napon

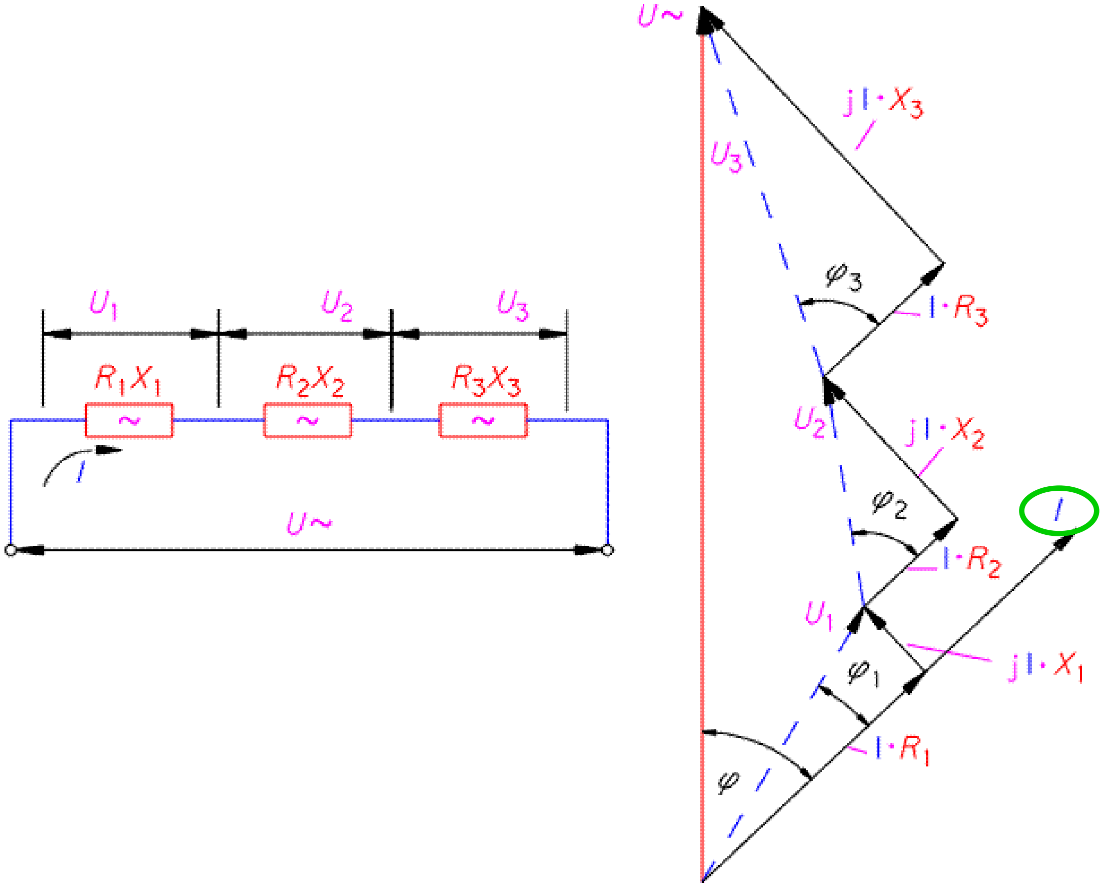


prema I Kirchhoffovom zakonu $\underline{I} = \sum I_R + j\sum I_L = I \cdot e^{-j\varphi}$

$$I = \sqrt{(I_1 \cdot \cos\varphi_1 + I_2 \cdot \cos\varphi_2)^2 + (I_1 \cdot \sin\varphi_1 + I_2 \cdot \sin\varphi_2)^2}$$

$$\varphi = \text{arc tg}\,\frac{I_1 \cdot \sin\varphi_1 + I_2 \cdot \sin\varphi_2}{I_1 \cdot \cos\varphi_1 + I_2 \cdot \cos\varphi_2}$$

Niz serijski spojenih impedancija



zajednička je struja

prema II Kirchhoffovom zakonu

$$\underline{U} = I\left(\sum_1^n R_i \pm j\sum_1^n X_i\right) = I \cdot Z \cdot e^{\pm j\varphi}$$

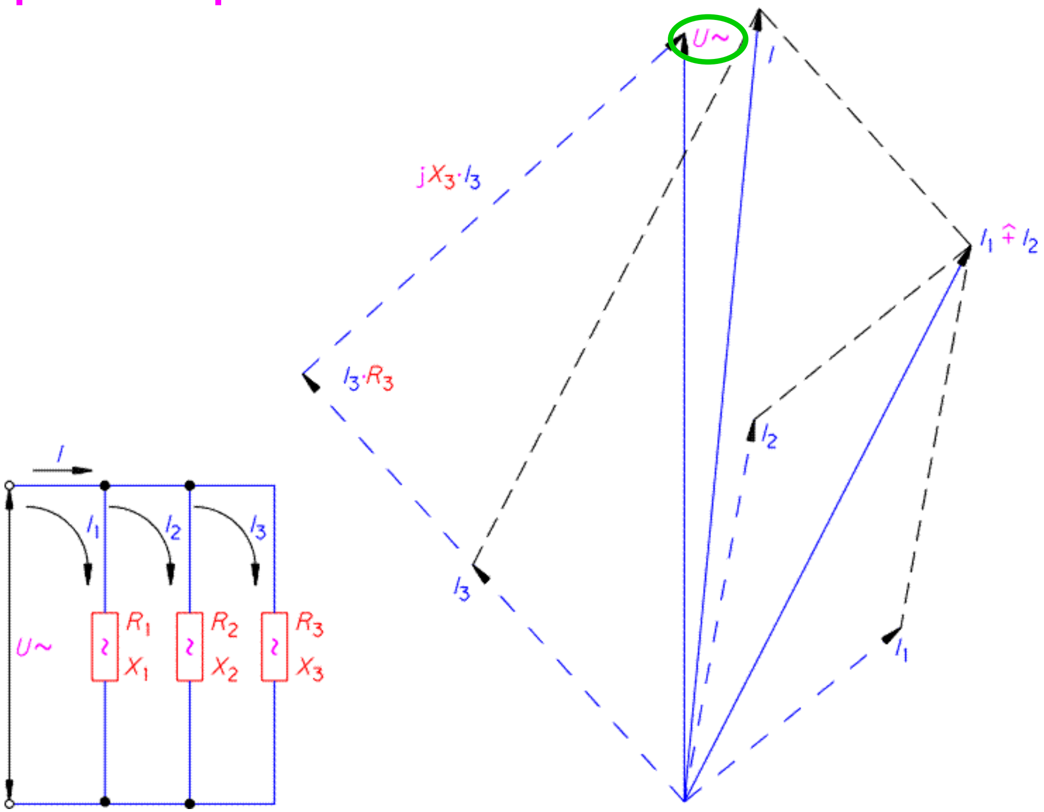
$$\underline{Z} = \sum_1^n R_i \pm j\sum_1^n X_i = Z \cdot e^{\pm j\varphi} = Z\angle\varphi$$

$$Z = \sqrt{\left(\sum_1^n R_i\right)^2 + \left(\sum_1^n X_i\right)^2}$$

$$\varphi = \text{atc tg}\frac{\sum_1^n X_i}{\sum_1^n R_i}$$

Niz paralelno spojenih impedancija

padovi napona elementata



zajednički je napon

prema I Kirchhoffovom zakonu

$$I_n = \frac{U}{\underline{Z}_n} = U \cdot \underline{Y}_n$$

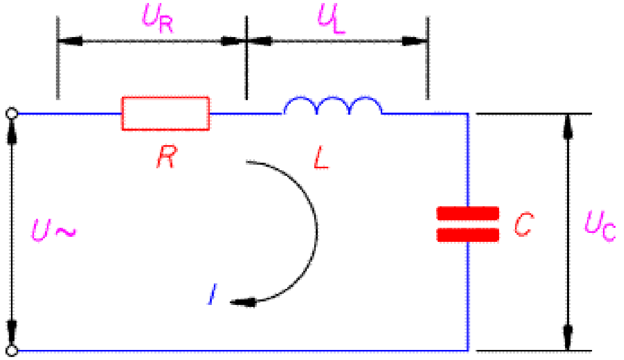
$$Y_n = \frac{1}{Z_n} = \frac{1}{R_n \pm jX_n} = Y_R \mp jY_X$$

$$\underline{Y}_n = \frac{1}{\underline{Z}_n} = \frac{1}{R_n + jX_n} = Y_{Rn} - jY_{Xn}$$

$$Y_{Rn} = \frac{R_n}{R_n^2 - X_n^2}$$

$$Y_{Xn} = \frac{X_n}{R_n^2 - X_n^2}$$

Serijski spoj R, L i C



$$u = L\frac{di}{dt} + i \cdot R + \frac{1}{C}\int i \cdot dt$$

$$i = \sqrt{2}I \cdot \sin \omega t$$

$$u = \sqrt{2}U \cdot \sin(\omega t + \varphi)$$

$$\underline{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right) = Z \cdot e^{j\varphi} = Z\angle\varphi$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

zajednička je struja

$$\varphi = \text{arc tg}\frac{\omega L - \frac{1}{\omega C}}{R}$$

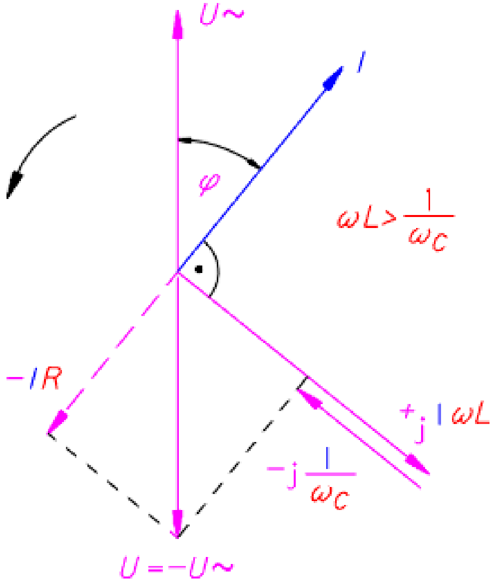
zbog preglednijeg prikaza

$$U_R = -I \cdot R$$

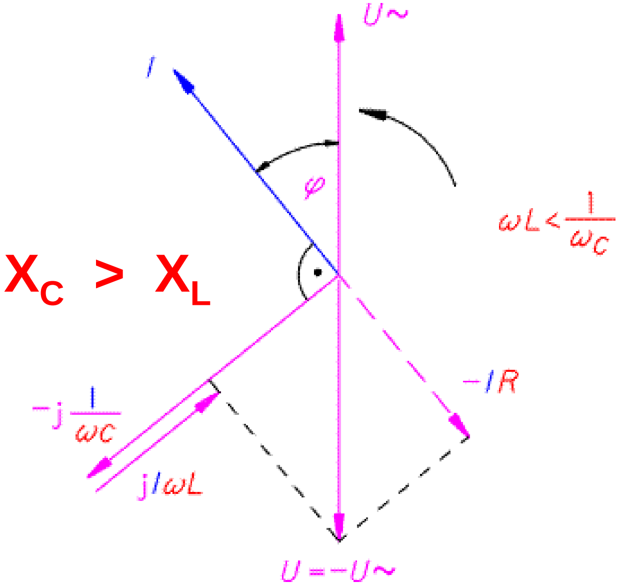
$$U_L = -jI \cdot \omega L$$

$$U_C = j\frac{I}{\omega C}$$

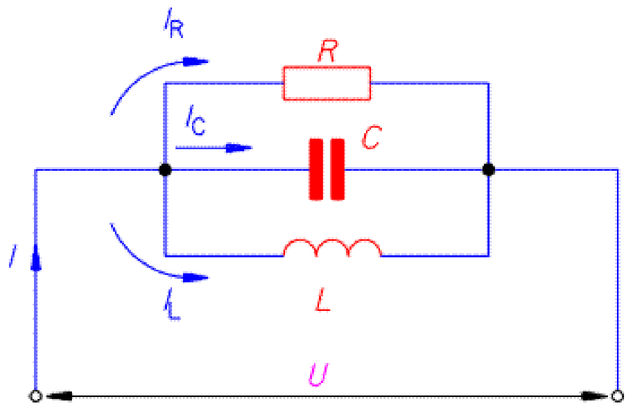
$X_L > X_C$



$X_C > X_L$



Paralelni spoj R, L i C

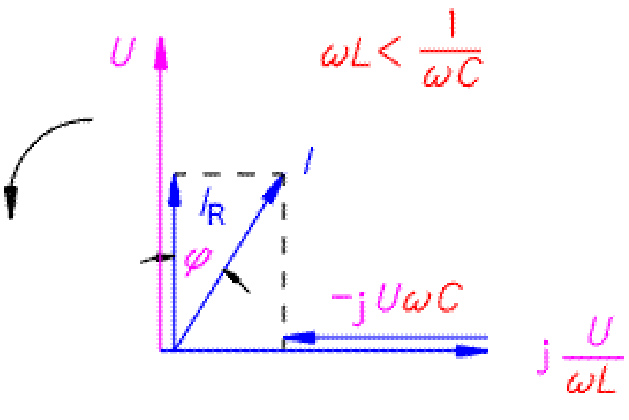


zajednički je napon

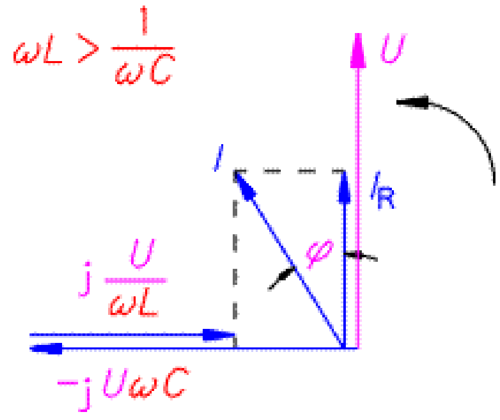
$$I_R = \frac{U}{R} \qquad I_L = \frac{U}{j\omega L} = -j\frac{U}{\omega L} \qquad I_C = U \cdot j\omega C = jU\omega C$$

$$\underline{Z} = \frac{R^2\omega^2L^2 + jR^2\omega L\left(1-\omega^2LC\right)}{R^2\left(1-\omega^2LC\right)^2 + \omega^2L^2} = Z \cdot e^{j\varphi} = Z\angle\varphi$$

$$Z = \frac{R\omega L}{\sqrt{R^2\left(1-\omega^2LC\right)^2 + \omega^2L^2}} \qquad \varphi = \text{arc tg}\frac{1-\omega^2LC}{\omega L}$$

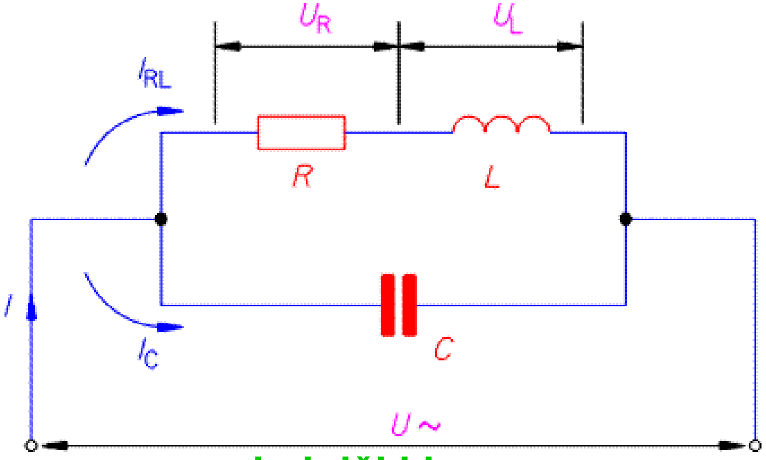


$X_L < X_C$



$X_C < X_L$

Serijska RL kombinacija paralelno spojena sa C



zajednički je napon

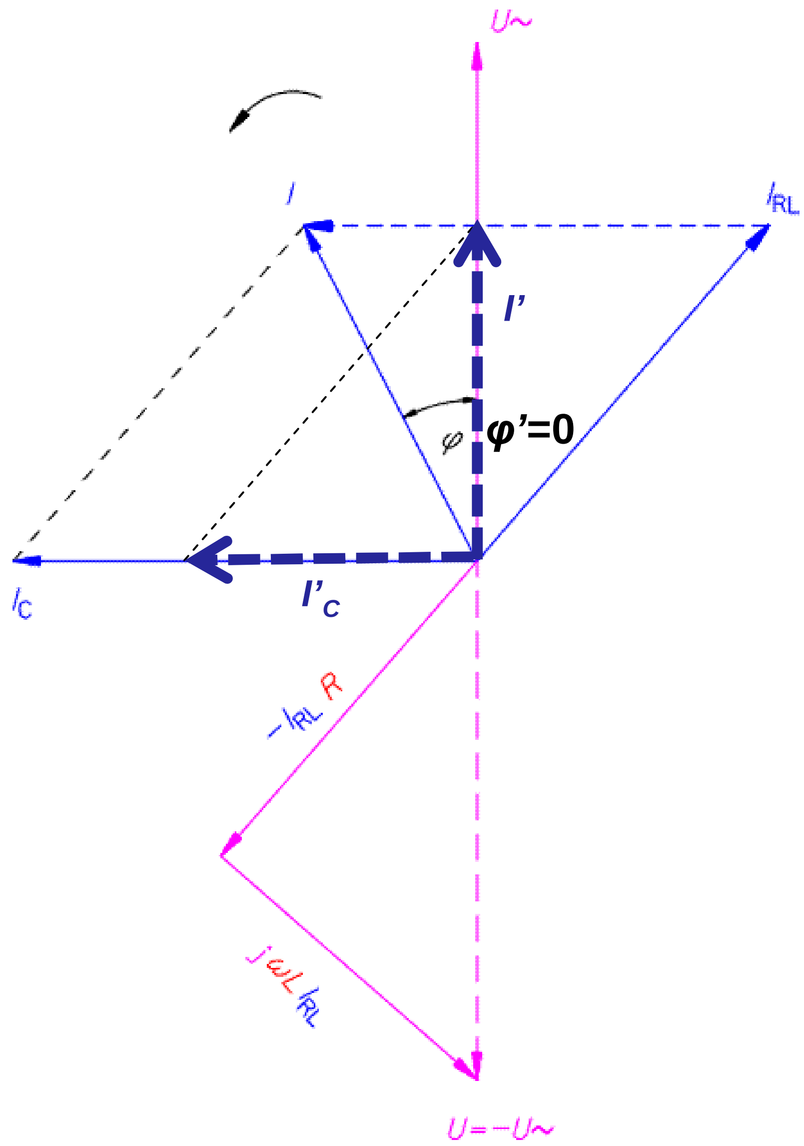
$$\underline{I}_{RL} = \frac{U}{R + j\omega L} \qquad \underline{U} = \underline{I}_{RL} \cdot R + j\underline{I}_{RL} \cdot \omega L = -U_{\sim}$$

$$\underline{I}_C = -jU \cdot \omega C$$

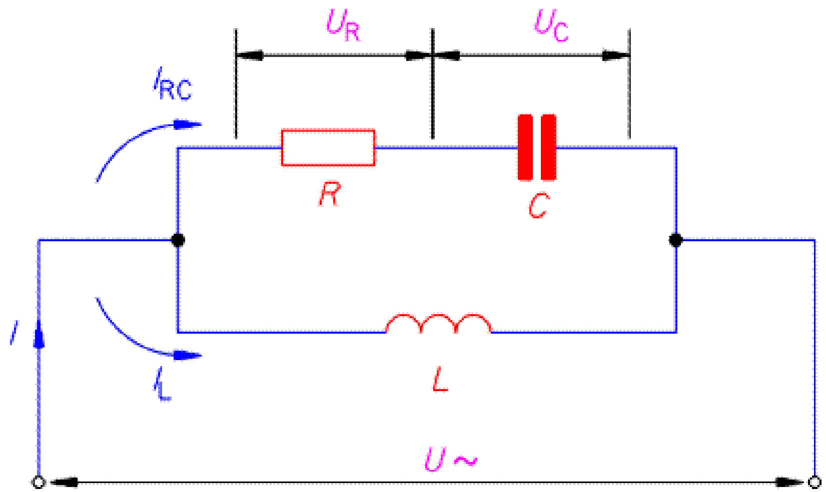
$$\underline{Z} = Z \cdot e^{j\varphi} = Z\angle\varphi$$

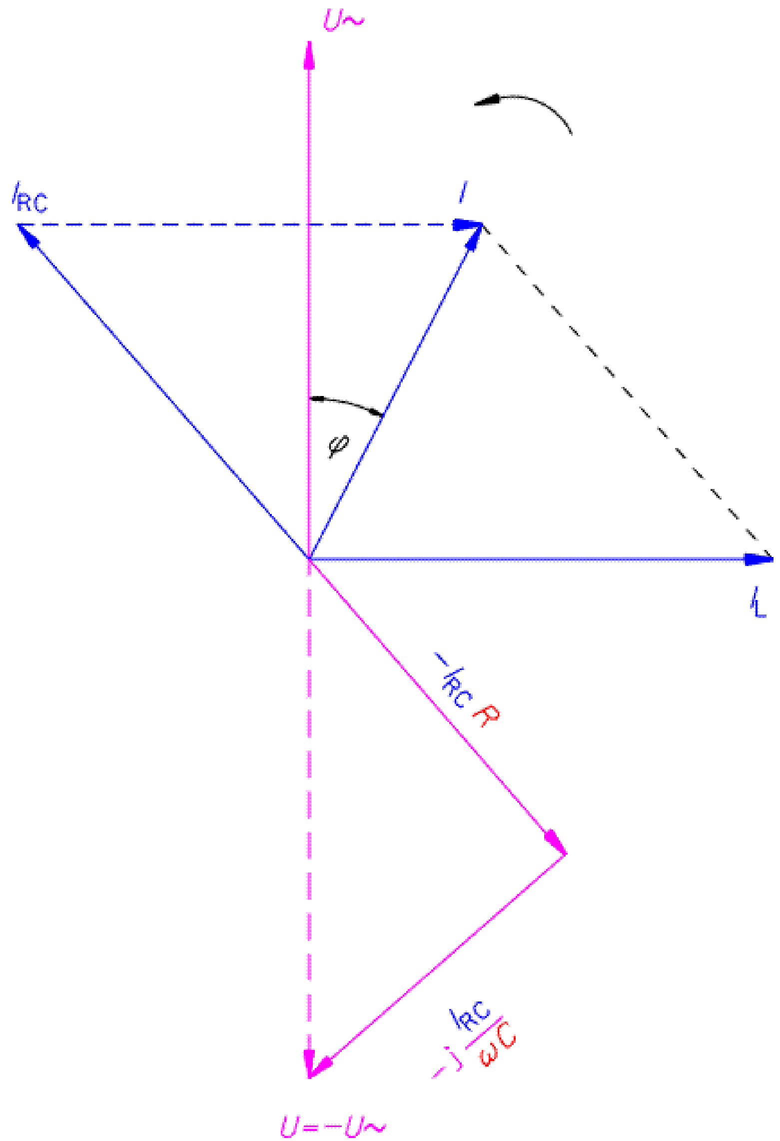
$$Z = \frac{\sqrt{R^2 + \omega^2 L^2}}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

$$\varphi = \text{arc tg}\,\frac{\omega\left[L - C\left(\omega^2 L^2 - R^2\right)\right]}{R}$$



Serijska RC kombinacija paralelno spojena sa L





zajednički je napon

$$\underline{U} = \underline{I}_{RC} \cdot R - j\frac{\underline{I}_{RC}}{\omega C} = -U_{\sim} \qquad \underline{I}_{RC} = \frac{U}{R - j\frac{1}{\omega C}}$$

$$\underline{I}_L = j\frac{U}{\omega L} \qquad \underline{Z} = Z \cdot e^{j\varphi} = Z\angle\varphi$$

$$Z = \frac{\omega L\sqrt{\omega^2 R^2 C^2 + 1}}{\sqrt{(\omega^2 LC - 1)^2 + \omega^2 R^2 C^2}} \qquad \varphi = \text{arc tg}\,\frac{\omega^2 LC + 1}{\omega^2 C^2 RL}$$

REZONANCIJA

nabijeni kondenzator - energija akumulirana u električnom polju

stalna struja kroz induktivitet - energija akumulirana u magnetskom polju

promjena struje – promjena akumuliranih energija – moguća razmjena energija

ritam razmjene energija - diktira izvor na kojeg su komponente priključene

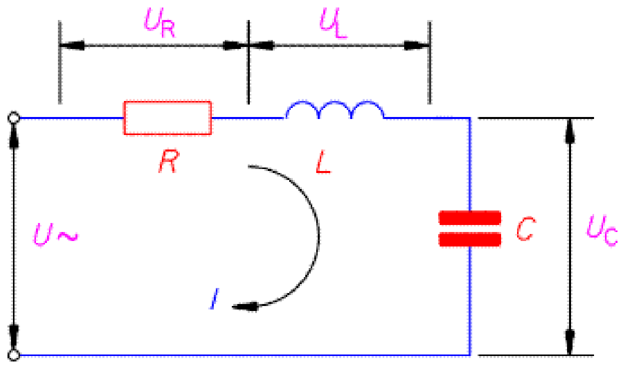
omski otpor - gušenje (otežavanje) razmjene energija

sinkronizacija ritma razmjene energija s frekvencijom izvora $\Rightarrow$ rezonancija

posljedice
- velike promjene vrijednosti rezultirajuće impedancije
- velike promjene jakosti struje kroz spoj
- velike promjene napona na komponentama spoja
- velike promjene faznog pomaka (predznaka) struje i napona

primjena
- izdvajanje željenih frekvencija
- prigušivanje neželjenih frekvencija

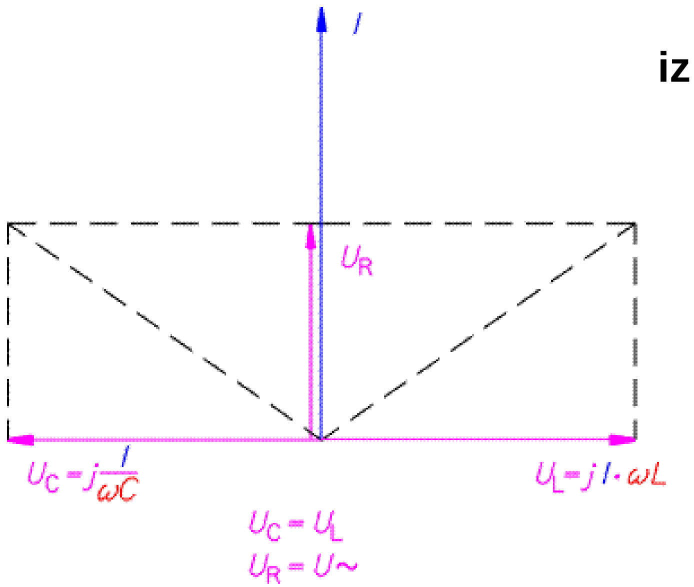
Serijska rezonancija



$$i \cdot R + L\frac{di}{dt} + \frac{1}{C}\int i \cdot dt = \sqrt{2}U \cdot \sin(\omega t)$$

$Z = R + j(X_L - X_C)$ **ako je** $\boxed{X_L = X_C}$

tada je $\boxed{Z = R}$



iz uvjeta $X_L = X_C$ **slijedi** $\omega L = \frac{1}{\omega C}$

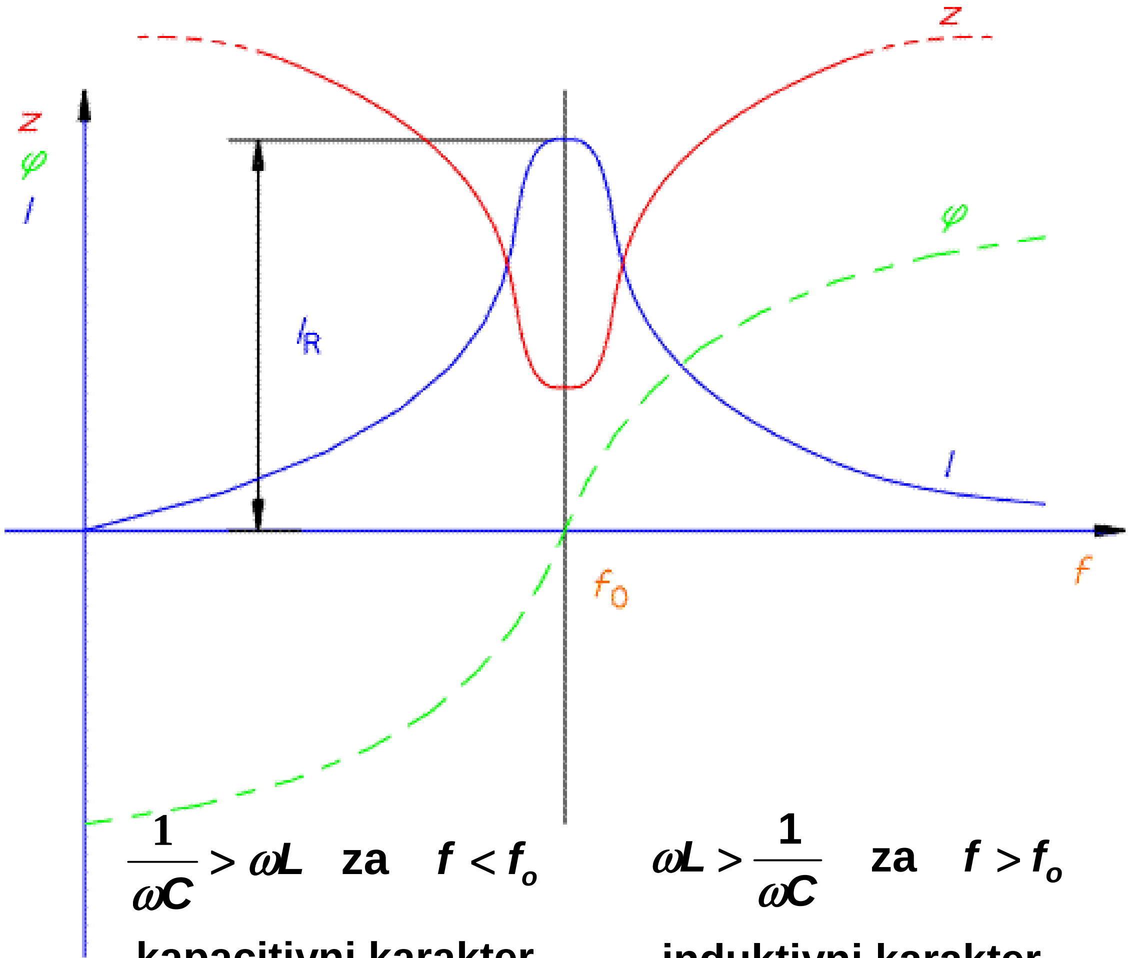
$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}} \qquad \boxed{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

rezonantna frekvencija

U_L **i** U_C **mogu biti >> od** U $\Rightarrow$ **mogući kvarovi**

Rezonancija je to izraženija što su gubici manji (otpor L i dielektrički gubici C) $\Rightarrow$

faktor dobrote titrajnog kruga $$k = \frac{\omega_o L}{R} = \frac{1}{R\omega_o C}$$



frekvencijska karakteristika serijskog rezonantnog kruga

$\frac{1}{\omega C} > \omega L$ za $f < f_o$

kapacitivni karakter

$\omega L > \frac{1}{\omega C}$ za $f > f_o$

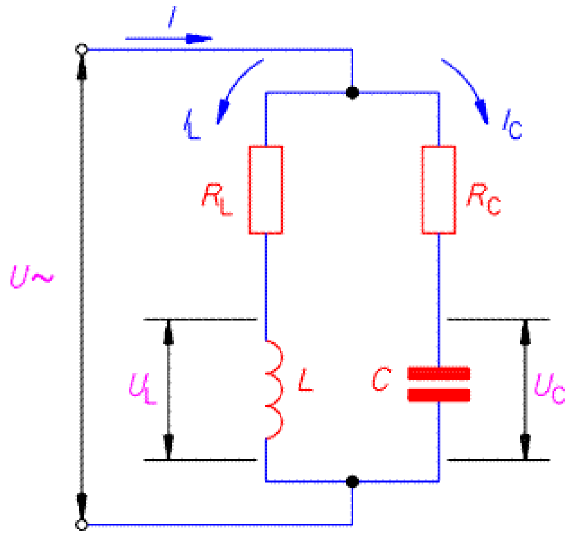
induktivni karakter

Paralelna rezonancija

$$\underline{I}_L = \frac{U}{R_L + j\omega L} \qquad \underline{I}_C = \frac{U}{R_C + \dfrac{j}{\omega\, C}}$$

imaginarni dio impedancije

$$\frac{\dfrac{1}{\omega C}}{R_C^2 + \dfrac{1}{\omega^2 C^2}} - \frac{\omega L}{R_L^2 + \omega^2 L^2} = 0$$



$$\omega_o = \left(\frac{R_L^2 - \dfrac{L}{C}}{LCR_C^2 - L^2} \right)^{\frac{1}{2}} = \frac{1}{\sqrt{LC}} \left(\frac{R_L^2 - \dfrac{L}{C}}{R_C^2 - \dfrac{L}{C}} \right)^{\frac{1}{2}}$$

rezonantna frekvencija

$$\underline{I}_o = U \frac{R_L + R_C \cdot \omega_o^2 LC}{R_L^2 + \omega_o^2 L^2}$$

struja pri rezonanciji

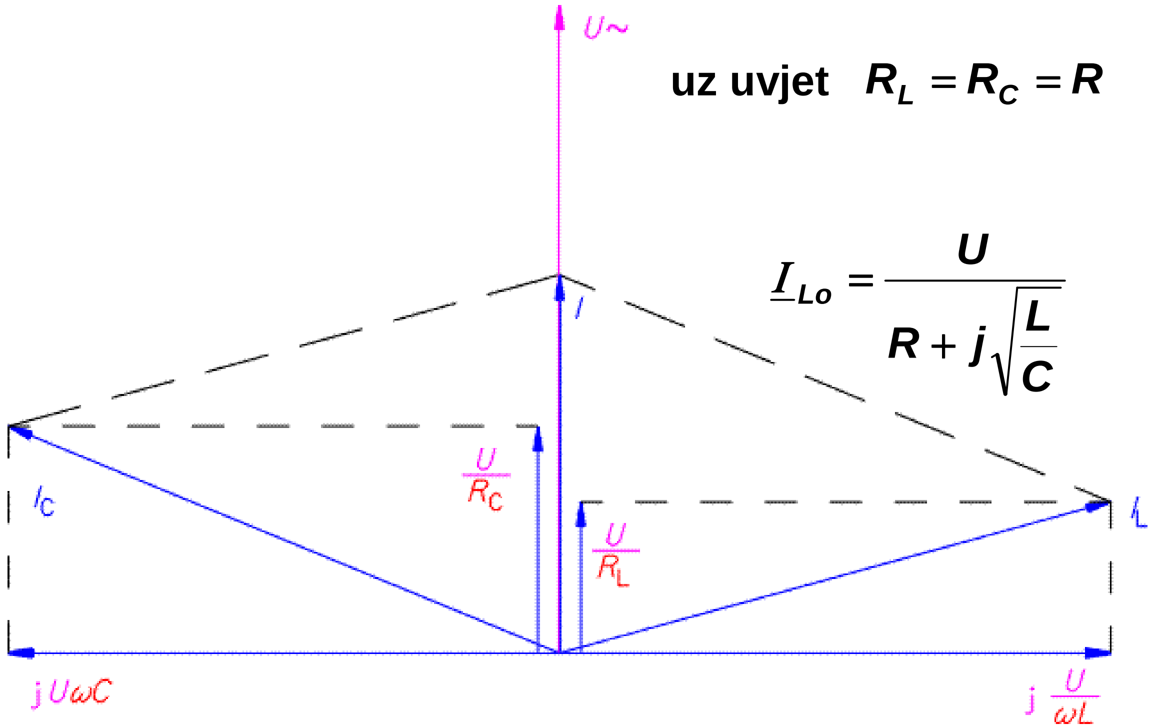
uz uvjet $R_L = R_C = R$ imamo dva rješenja

$\omega_o L = \dfrac{1}{\omega_o C}$ odakle izlazi $\omega_o = \dfrac{1}{\sqrt{LC}}$

kao kod serijskog titrajnog kruga

$R = \sqrt{\dfrac{L}{C}}$ što odgovara valnom otporu kruga

otporna rezonancija neovisna o frekvenciji



uz uvjet $R_L = R_C = R$ imamo $\underline{I}_{Co} = \dfrac{U}{R - j\sqrt{\dfrac{L}{C}}}$ i

$$\underline{I}_{Lo} = \frac{U}{R + j\sqrt{\dfrac{L}{C}}}$$

te $Z_o = \dfrac{R^2 + \dfrac{L}{C}}{2R}$

impedancija paralelnog kruga u rezonanciji
valni otpor
valna impedancija
rezonancijska impedancija

frekvencijska rezonacija $L \to 0$ ili $C \to \infty$ slijedi $Z \to \dfrac{R}{2}$ $C \to 0$ ili $L \to \infty$ slijedi $Z_o \to \infty$

otporna rezonacija $R^2 = \dfrac{L}{C}$ slijedi $Z_o = R$ ili $Z_o = \sqrt{\dfrac{L}{C}}$

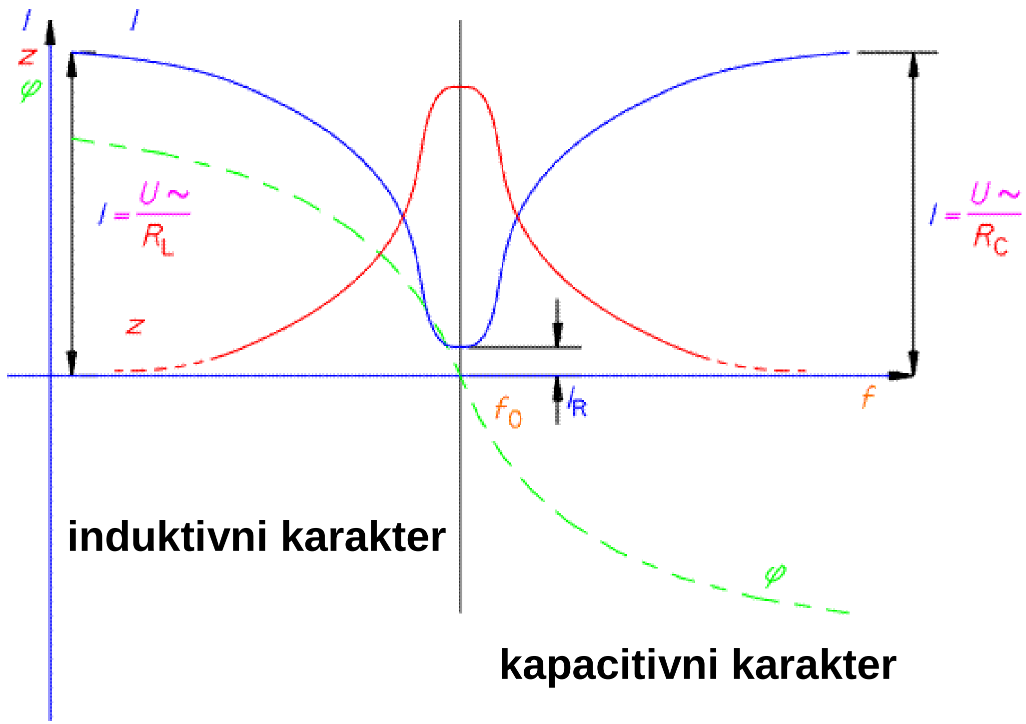
maksimalna pri $Z_o = \dfrac{R}{2}$ $(L \to 0$ ili $C \to \infty)$ $I_o = \dfrac{2U}{R}$

minimalna pri $Z_o \to \infty$ $(C \to 0$ ili $R \to 0)$ $I_o \to 0$

u realnim uvjetima

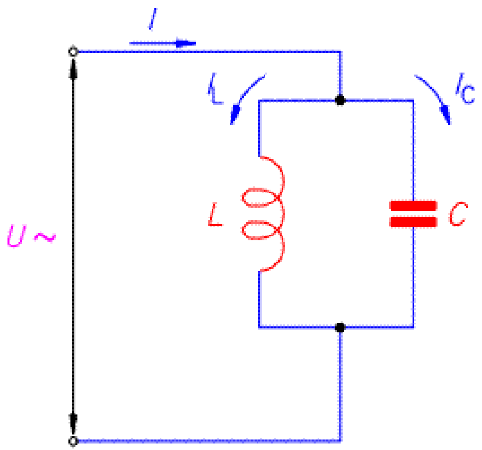
$$I_o = \frac{U}{R} = \frac{U}{\sqrt{\dfrac{L}{C}}}$$

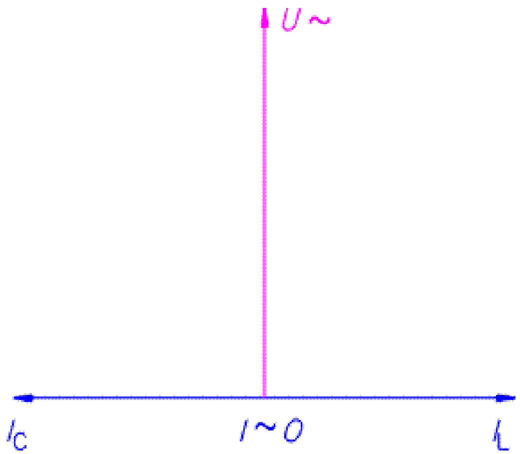
$$U = I_o \cdot \sqrt{\frac{L}{C}}$$



frekvencijska karakteristika paralelnog rezonantnog kruga

ako nema otpornih komponenti titrajni krug prema vani djeluje kao izolator ali unutar njega teku vrlo velike struje





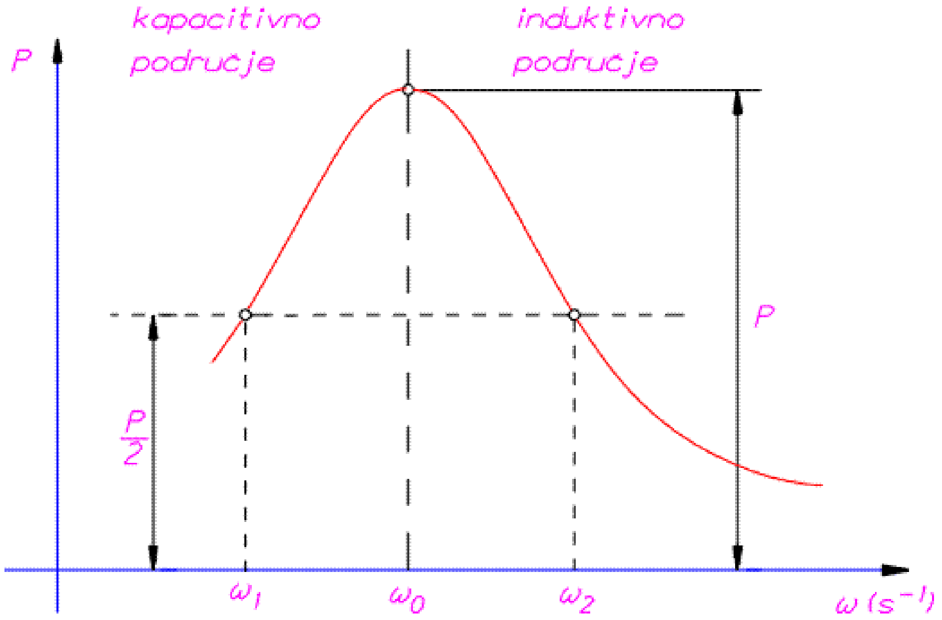
paralelni titrajni krug sastavljen od čistih reaktivnih komponenti

Rezonancijski strujni krugovi

faktor kvalitete kruga $Q = \frac{U_L}{U} = \frac{U_C}{U} = \frac{\omega_o L}{R} = \frac{1}{\omega_o C R}$ **općenito** $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

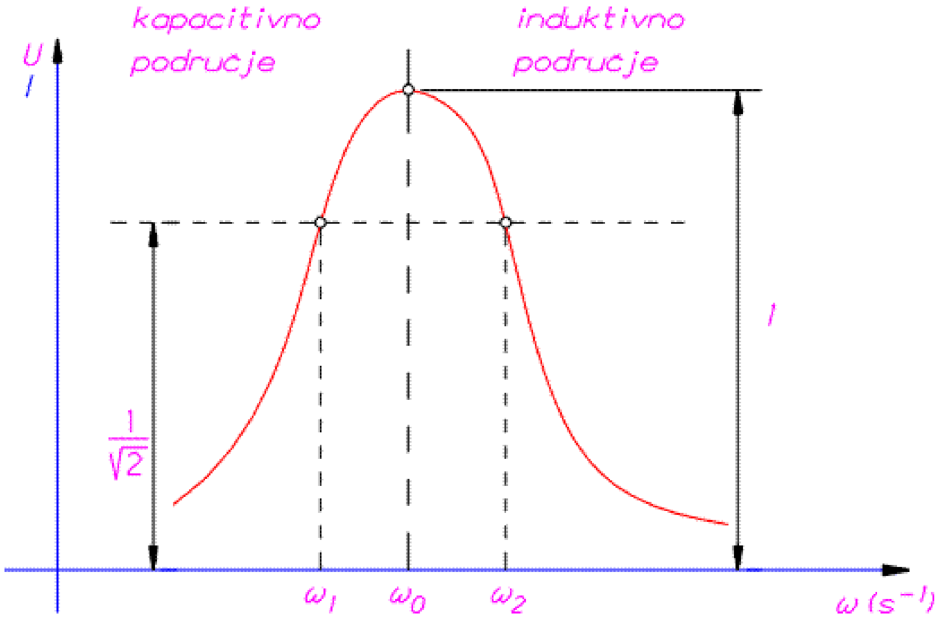
izraženost rezonanacije opisuje se pomoću širine pojasa $\Delta\omega$

snaga



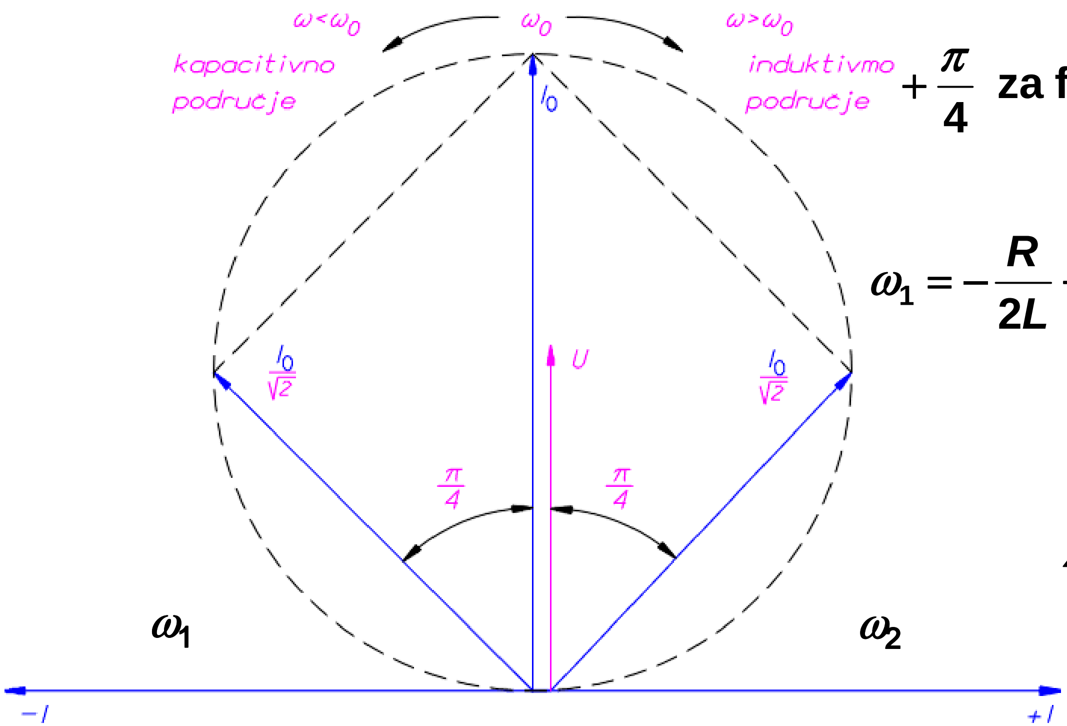
$\Delta\omega = \omega_2 - \omega_1$

napon ili struju



$\Delta\omega = \omega_2 - \omega_1$

za serijski krug



granica - struja manja za $\sqrt{2}$

$+\frac{\pi}{4}$ **za frekvenciju** ω_1 **i** $-\frac{\pi}{4}$ **za frekvenciju** ω_2

granične frekvencije

$$\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L}$$ **širina pojasa**

$$\Delta f = f_2 - f_1 = \frac{1}{2\pi}(\omega_2 - \omega_1) = \frac{1}{2\pi} \cdot \frac{R}{L} = f_o \cdot k$$

širina pojasa izražena promjenom C

$$C_2 - C_1 = \frac{2C_o}{k} = 2 \cdot C_o \frac{R}{\omega L}$$

C_o **kapacitet pri rezonanciji uz** $\omega = \omega_o$

C_2 **kapacitet pri frekvenciji** ω_2

C_1 **kapacitet pri frekvenciji** ω_1

širina pojasa izražena promjenom L

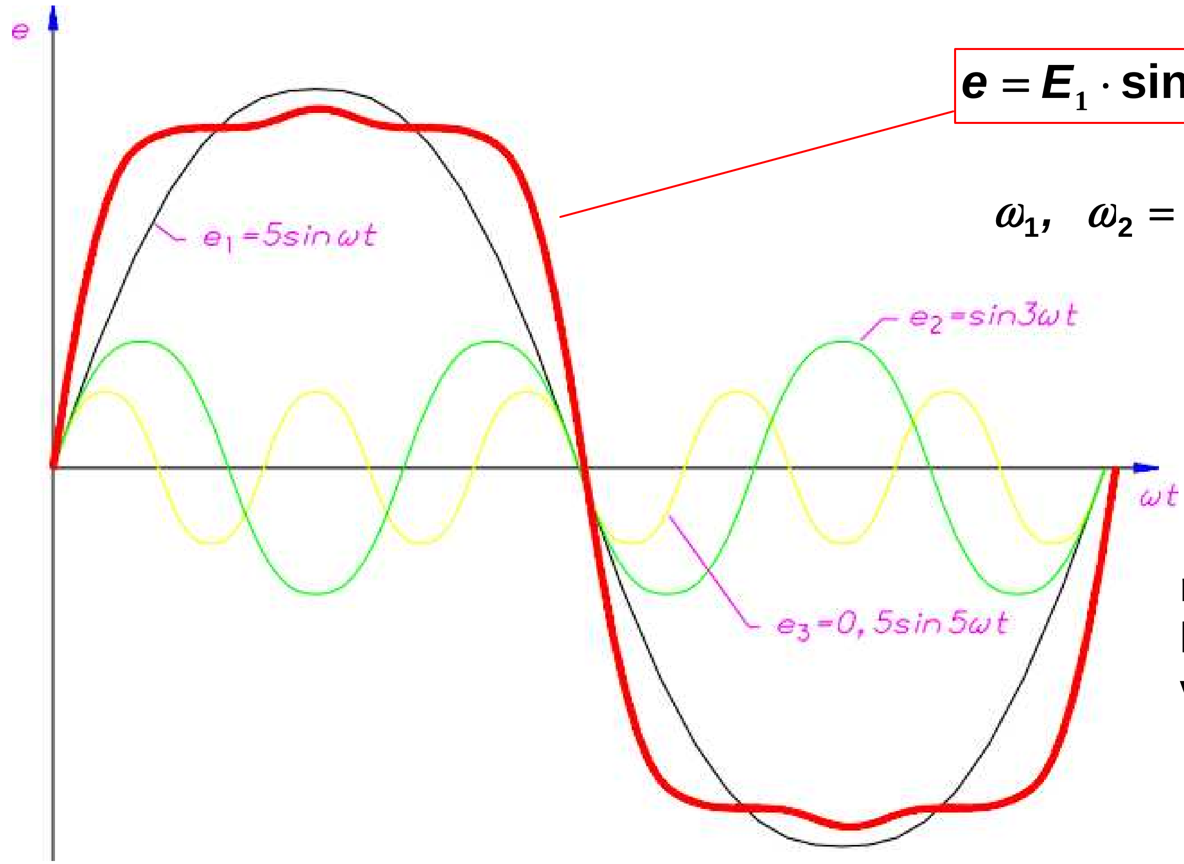
$$L_2 - L_1 = \frac{2L_o}{k} = 2L_o R\omega C$$

L_o **induktivitet pri rezonanciji uz** $\omega = \omega_o$

L_2 **induktivitet pri frekvenciji** ω_2

L_1 **induktivitet pri frekvenciji** ω_1

Miješanje i izdvajanje stuja različite frekvencije



$$e = E_1 \cdot \sin\omega_1 t + E_2 \cdot \sin\omega_2 t + E_3 \cdot \sin\omega_3 t$$

$\omega_1, \quad \omega_2 = 3 \cdot \omega_1, \quad \omega_3 = 5 \cdot \omega_1$ **frekvencije**

$\omega_1 = \omega$

E_1=5, E_2=1, E_3=0,5 - amplitude

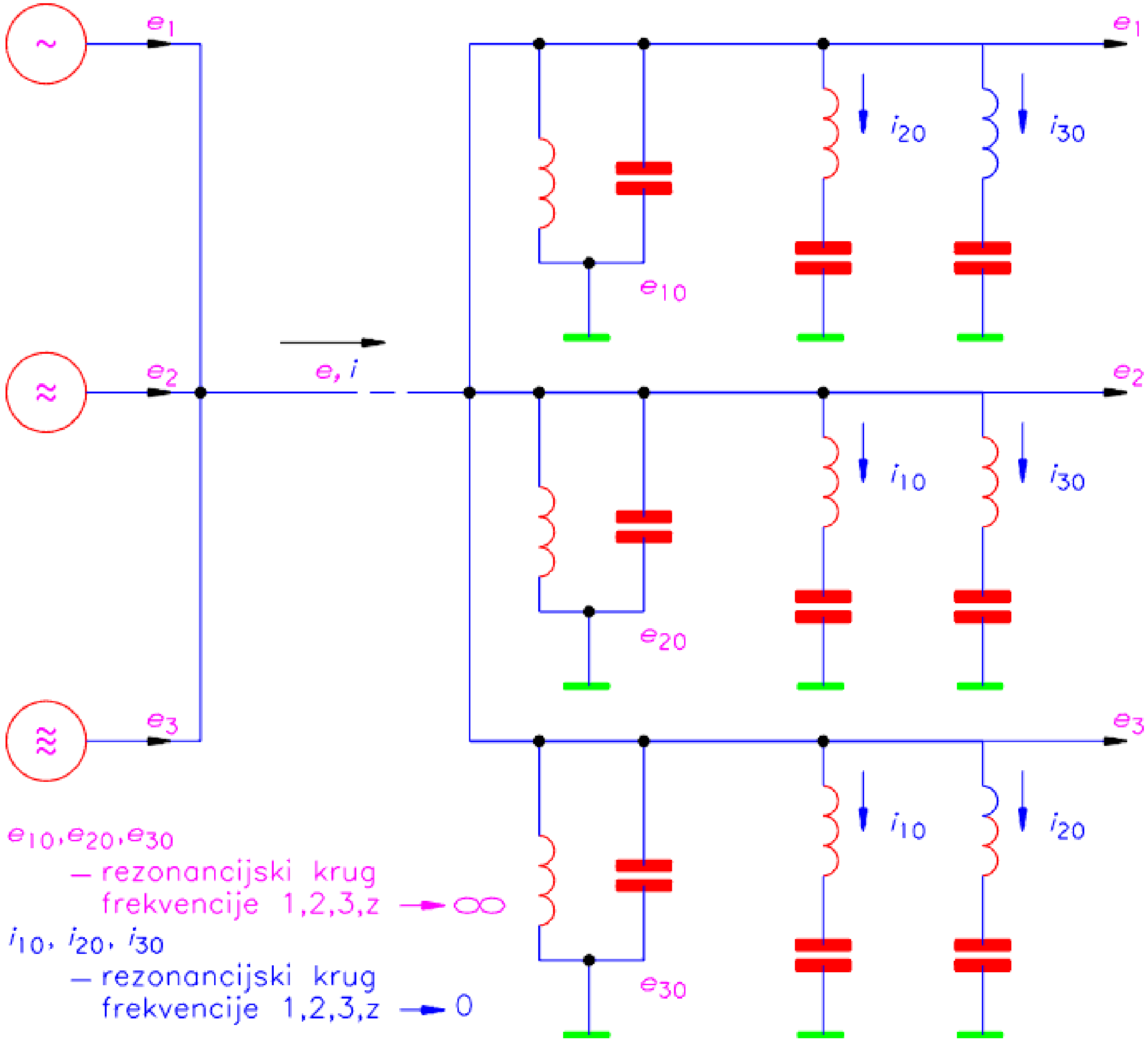
nesinusna veličina sadrži više harmonike - parne i neparne višekratnike osnovne frekvencije

odstupanje od sinusnog valnog oblika

tjemeni faktor $\frac{I_{max}}{I_{ef}} = \sqrt{2} = 1{,}41$

faktor oblika $\frac{I_{ef}}{I_{sr}} = \frac{I_{max} \cdot \pi}{\sqrt{2} \cdot 2 \cdot I_{max}} = \frac{\pi}{2\sqrt{2}} = \frac{\pi}{\sqrt{8}} = 1{,}11$

za sinusni valni oblik



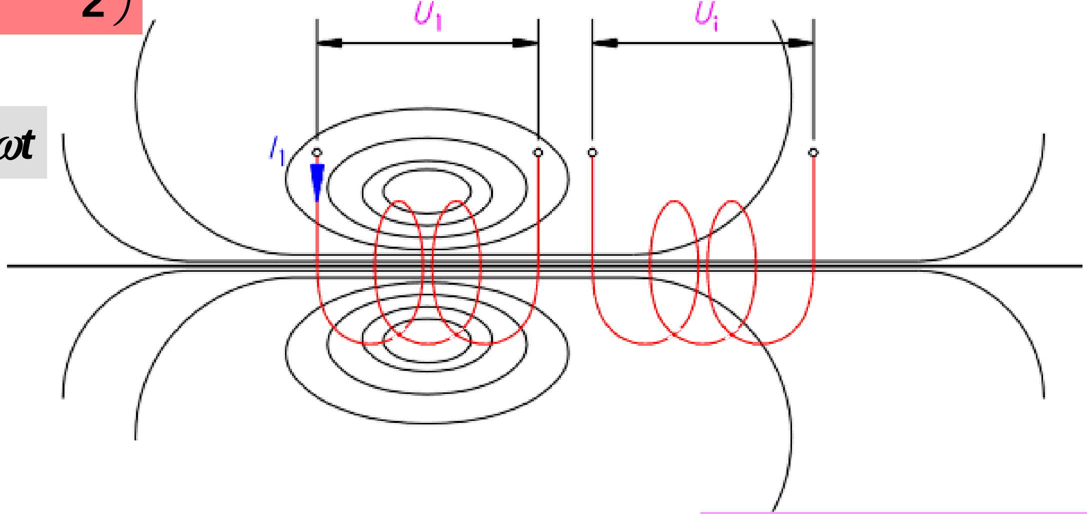
e1
e2
e3
e, i
e1
e2
e3
i20
i30
i10
i30
i10
i20
e10
e20
e30
e10,e20,e30
– rezonancijski krug
frekvencije 1,2,3,z → ∞
i10, i20, i30
– rezonancijski krug
frekvencije 1,2,3,z → 0

INDUCIRANI NAPON IZMJENIČNE STRUJE

$$u_1 = \sqrt{2} \cdot U_1 \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

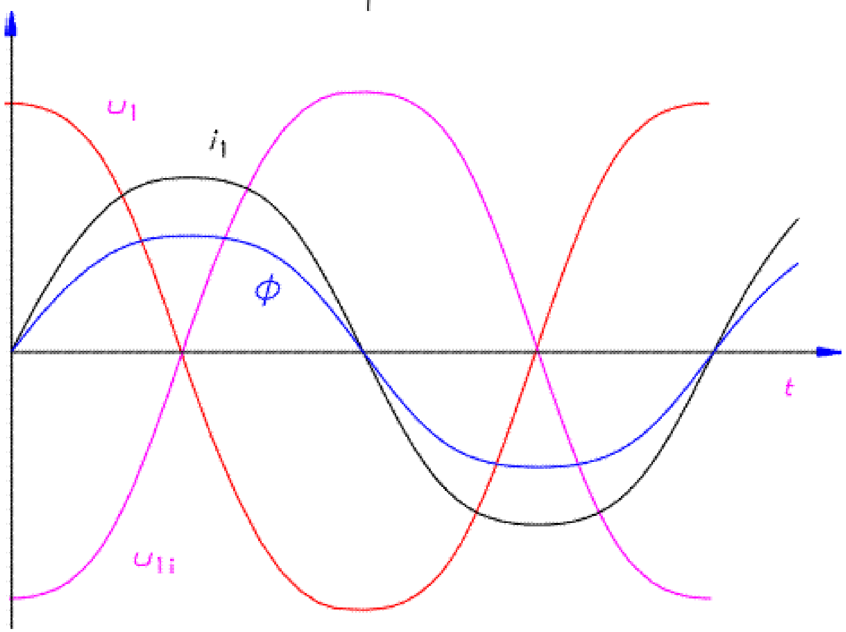
napon izvora

inducirani napon



$$i_1 = \sqrt{2} \cdot I_1 \cdot \sin \omega t$$

$$\Phi = \sqrt{2} \cdot \Phi_1 \cdot \sin \omega t$$



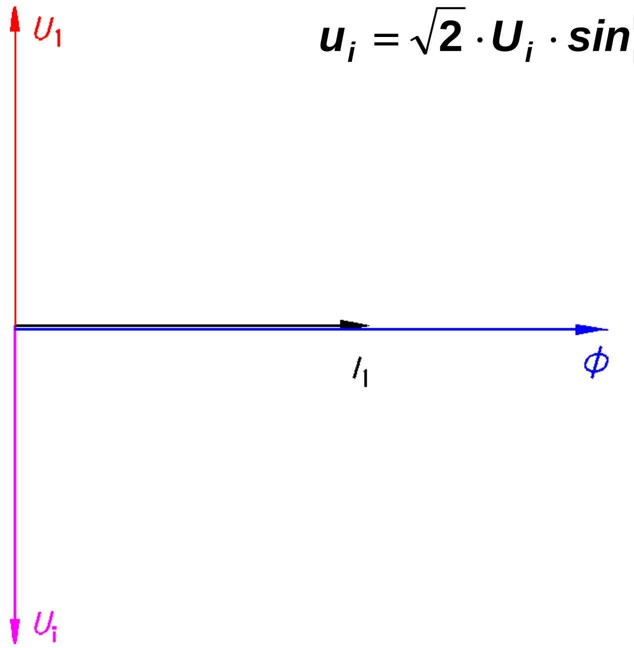
$$u_i = -\frac{d\Phi}{dt} = -\Phi_{max} \cdot \frac{d(\sin \omega t)}{dt}$$

$$= \omega \cdot \Phi_{max} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$U_i = \frac{\omega \cdot \Phi_{max}}{\sqrt{2}} = \omega L \cdot I_1$$ **efektivna vrijednost induciranog napona ako je Φ zavojnica zajednički (jednak)**

$$u_1 = \sqrt{2} \cdot U_1 \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$ **napon izvora**

$$u_i = \sqrt{2} \cdot U_i \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$$ **inducirani napon**



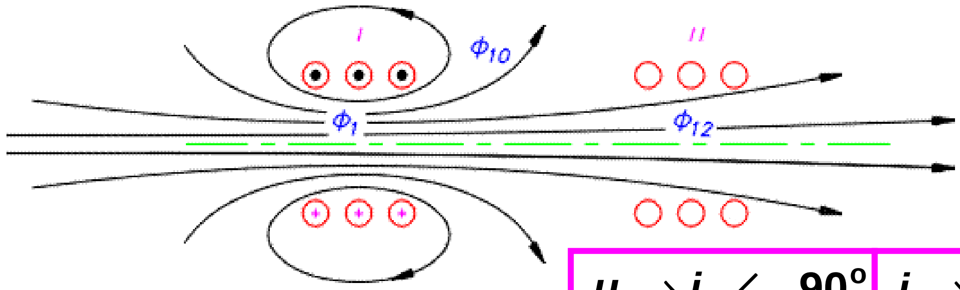
$$+\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) = \pi$$ **fazna razlika (protufaza)**

$$\underline{I}_1 = I_1 \angle 0^o$$

$$\underline{U}_1 = U_1 \angle +90^o$$

$$\underline{U}_i = U_i \angle -90^o$$

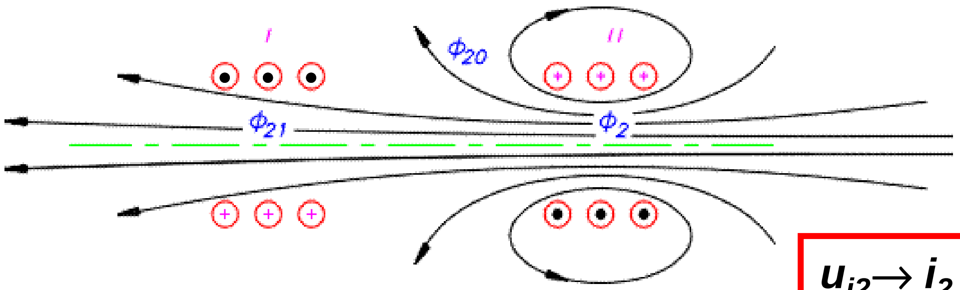
Međuinduktivitet



otvoren strujni krug **II** svitka

$u_1 \rightarrow i_1$ kroz **I** svitak $\rightarrow \Phi_1 \rightarrow \Phi_{10} + \Phi_{12}$
$\rightarrow u_{i2}$ **II** svitka

$u_1 \rightarrow i_1 \angle -90^o$	$i_1 \rightarrow \Phi_{12} \angle 0^o$	$\Phi_{12} \rightarrow u_{i2} \angle -90^o$	$u_1 \rightarrow u_{i2} \angle -180^o$



zatvoren strujni krug **II** svitka

i_2 kroz **II** svitak $\rightarrow \Phi_2 \rightarrow \Phi_{20} + \Phi_{21}$
$\rightarrow u_{1i}$ **I** svitka

$u_{i2} \rightarrow i_2 \angle -90^o$	$i_2 \rightarrow \Phi_{21} \angle 0^o$	$\Phi_1 \rightarrow \Phi_{21} \angle -180^o$

$$u_1 = N_1 \frac{d\Phi_1}{dt} \qquad u_{i1} = -N_1 \frac{d\Phi_{21}}{dt} \qquad u_{i2} = -N_2 \frac{d\Phi_{12}}{dt} \qquad u_2 = -N_2 \frac{d\Phi_2}{dt}$$

u ravnoteži $\Phi_{12} = \Phi_{21}$

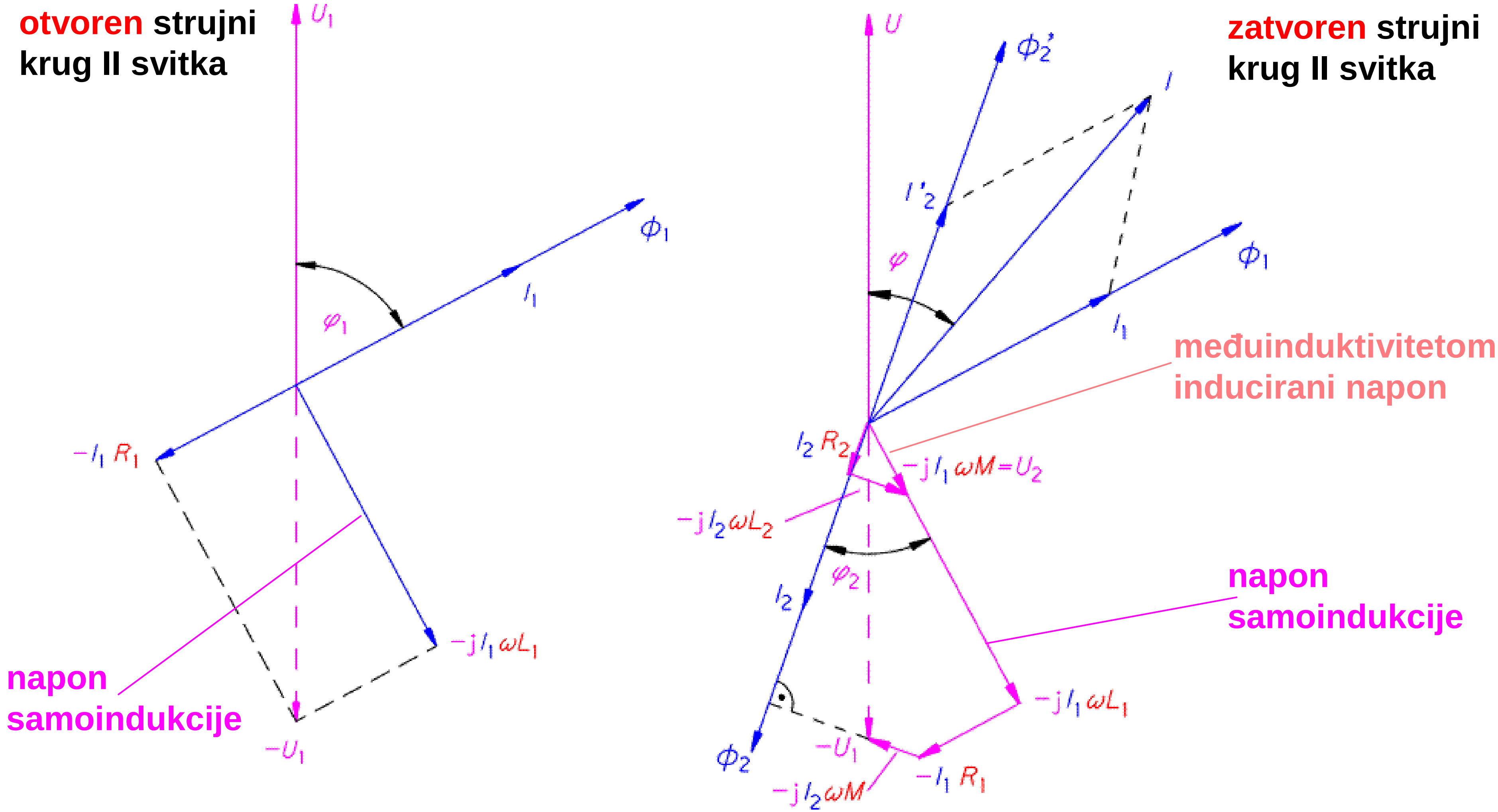
$$N_2 \cdot \frac{d\Phi_{12}}{di_1} = N_1 \frac{d\Phi_{21}}{di_2} = M$$

MEĐUINDUKTIVITET

$$u_{i1} = -M \frac{di_2}{dt} \qquad u_{i2} = -M \frac{di_1}{dt}$$

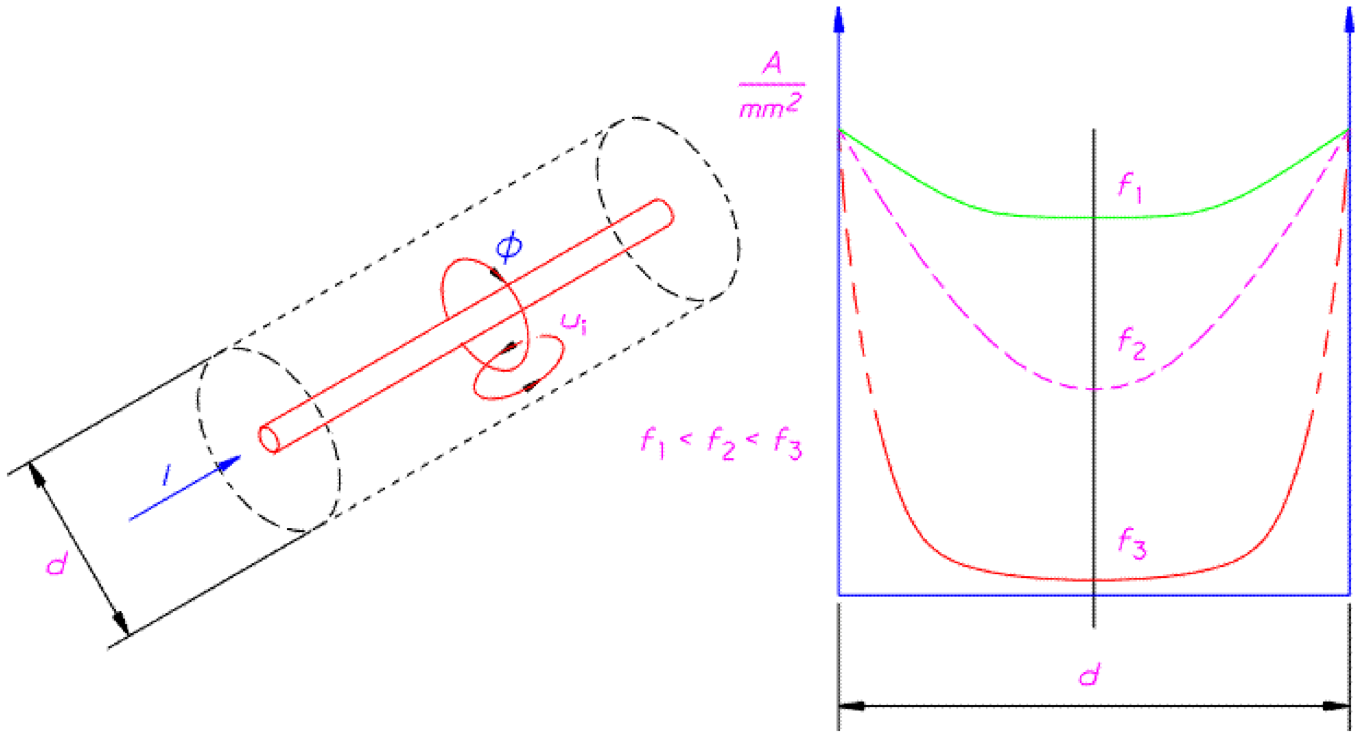
za zrak vrijedi $M = N_2 \dfrac{\Phi_{12}}{I_2} = N_1 \dfrac{\Phi_{21}}{I_1}$

efektivna vrijednost napona $u_{i2} = I_1 \cdot \omega M$



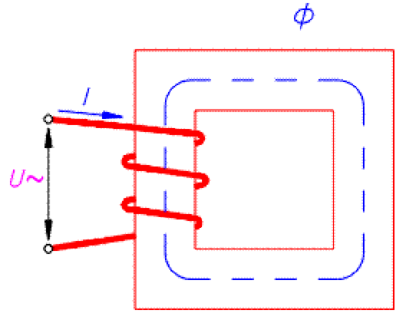
trenutne vrijednosti napona u pojedinom svitku

$$u_1 = i_1 \cdot R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad u_2 = M \frac{di_1}{dt} = i_2 \cdot R_2 + L_2 \frac{di_2}{dt}$$



skin efekt ⇒ potiskivanje struje prema vanjskom dijelu vodiča
⇒ manja površina ⇒ veći otpor
Za visoke frekvencije ⇒ valovodi umjesto vodiča

MAGNETSKI VODLJIVI MATERIJALI U IZMJENIČNOM MAGNETNOM POLJU



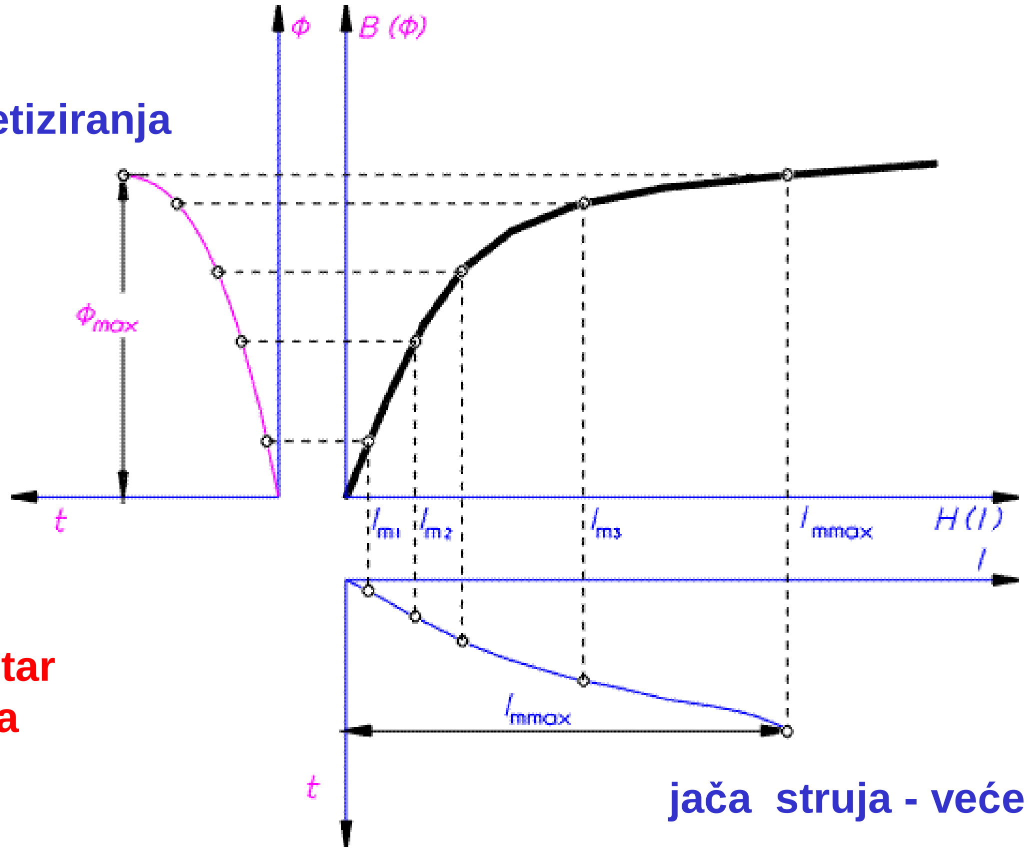
veće magnetne indukcije i bolje magnetne sprege

svojstva Fe određena primjesama i tehnološkim postupkom obrade

zasićenje i struja magnetiziranja

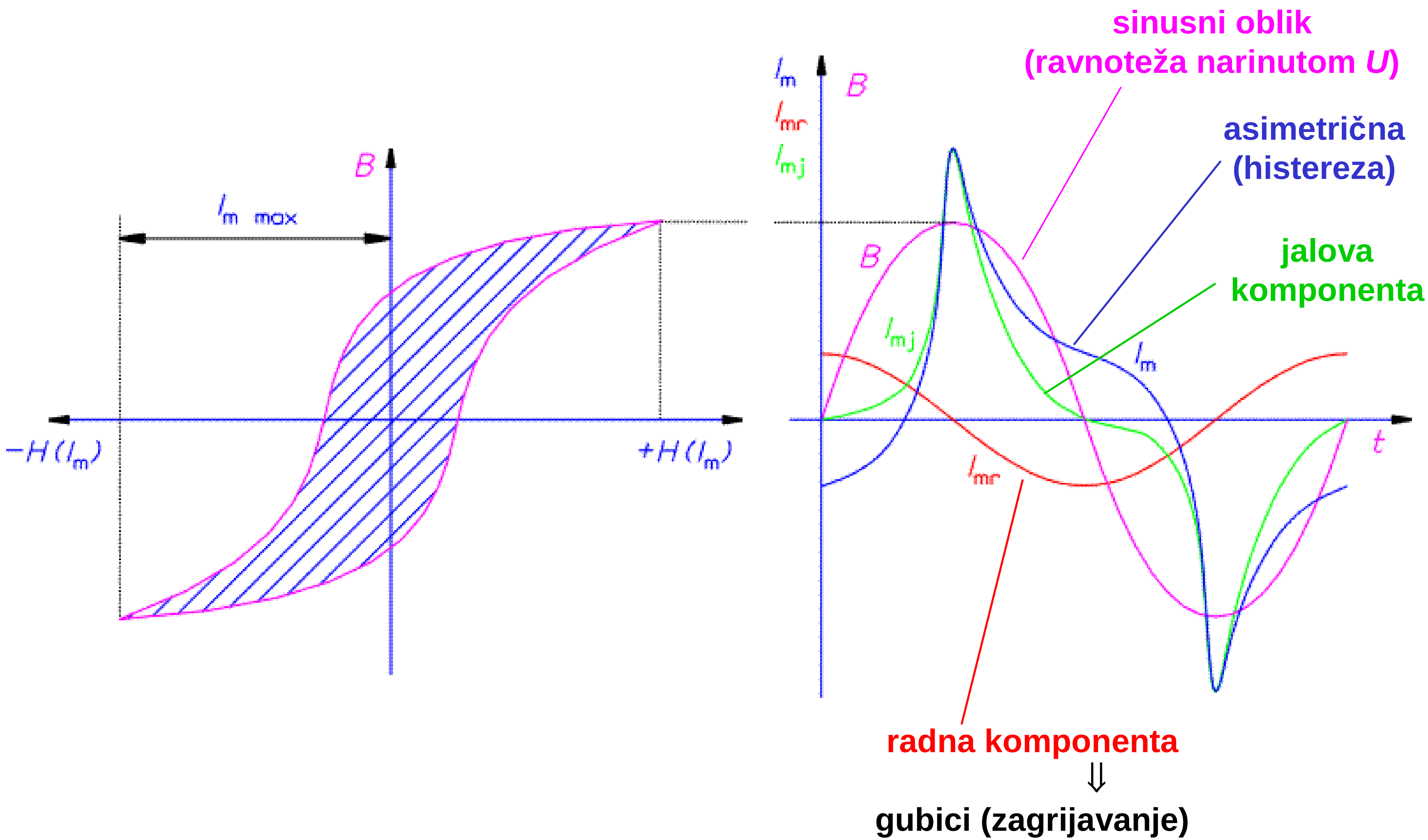
Φ sinusnog oblika zbog suprotstavljanja narinutom naponu

bez izobličenja unutar linearnog područja



jača struja - veće izobličenje

histereza i struja magnetiziranja



gubici zbog histereze

$$W = V\int_0^B H \cdot dB$$

V - volumen magnetiziranja
B - maksimalna magnetna indukcija
H - jakost magnetnog polja

uz površinu histereze =0 $\int_0^B H \cdot dB \to 0$ mekomagnetni materijali sa što užom krivuljom

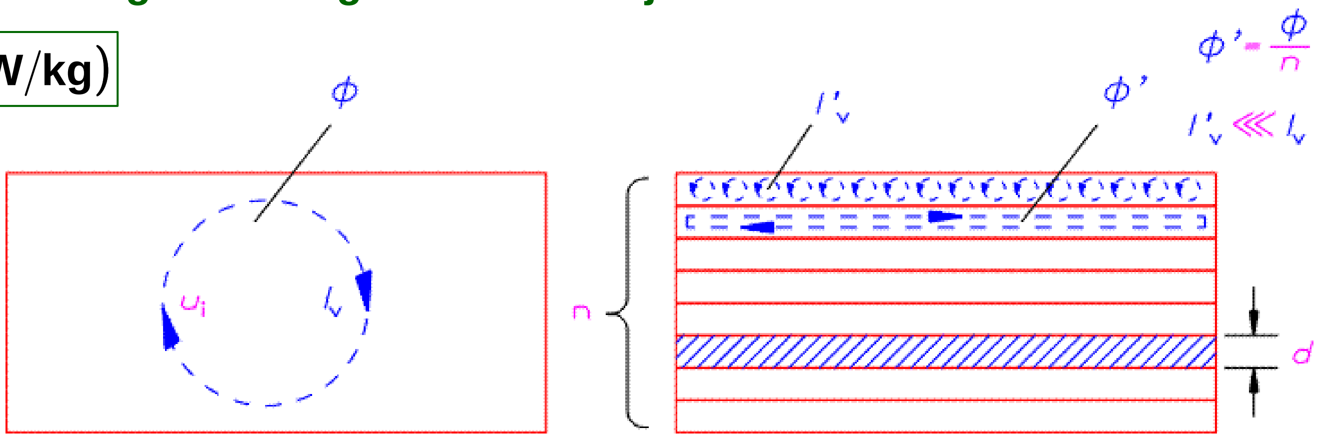
$$w_h = k_h \cdot f \cdot B_{max}^{\alpha} \quad \text{(W/kg)}$$

k_h - koeficijent linearno ovisan o kvaliteti materijala
f - frekvencija magnetiziranja
B_{max} - maksimalna magnetna indukcija
α - eksponent nelinearne ovisnosti o magnetnoj indukciji, ovisan o magnetnoj indukciji, (vrijednost 1,6 do 2)

gubici zbog vrtložnih struja

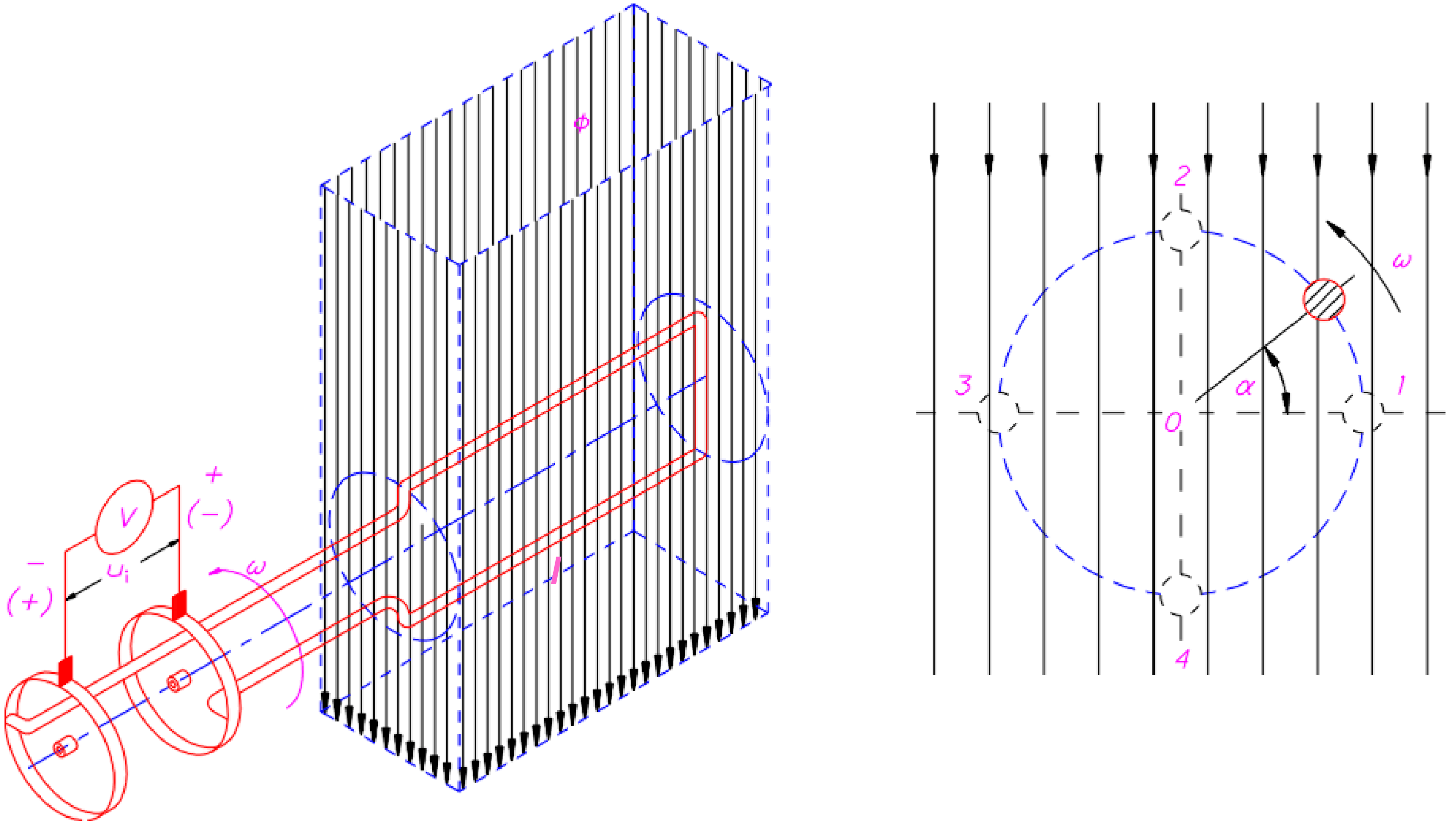
$$w_v = k_v \cdot f^2 \cdot B_{max}^2 \cdot d^2 \quad \text{(W/kg)}$$

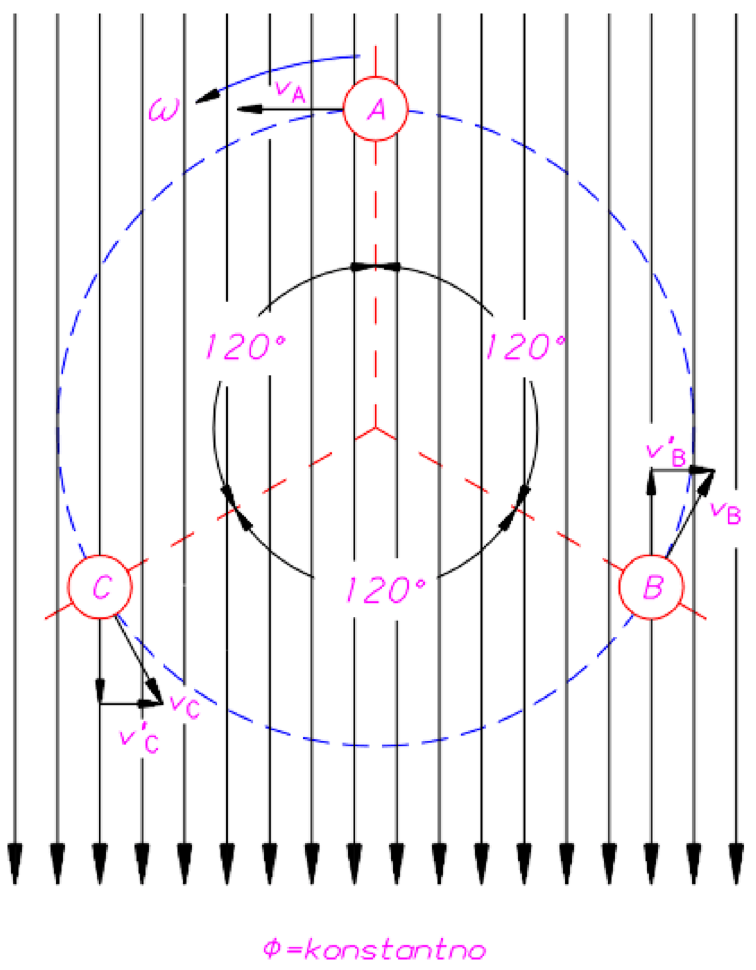
d -debljina lima u mm
k_v - koeficijent linearno ovisan o kvaliteti materijala



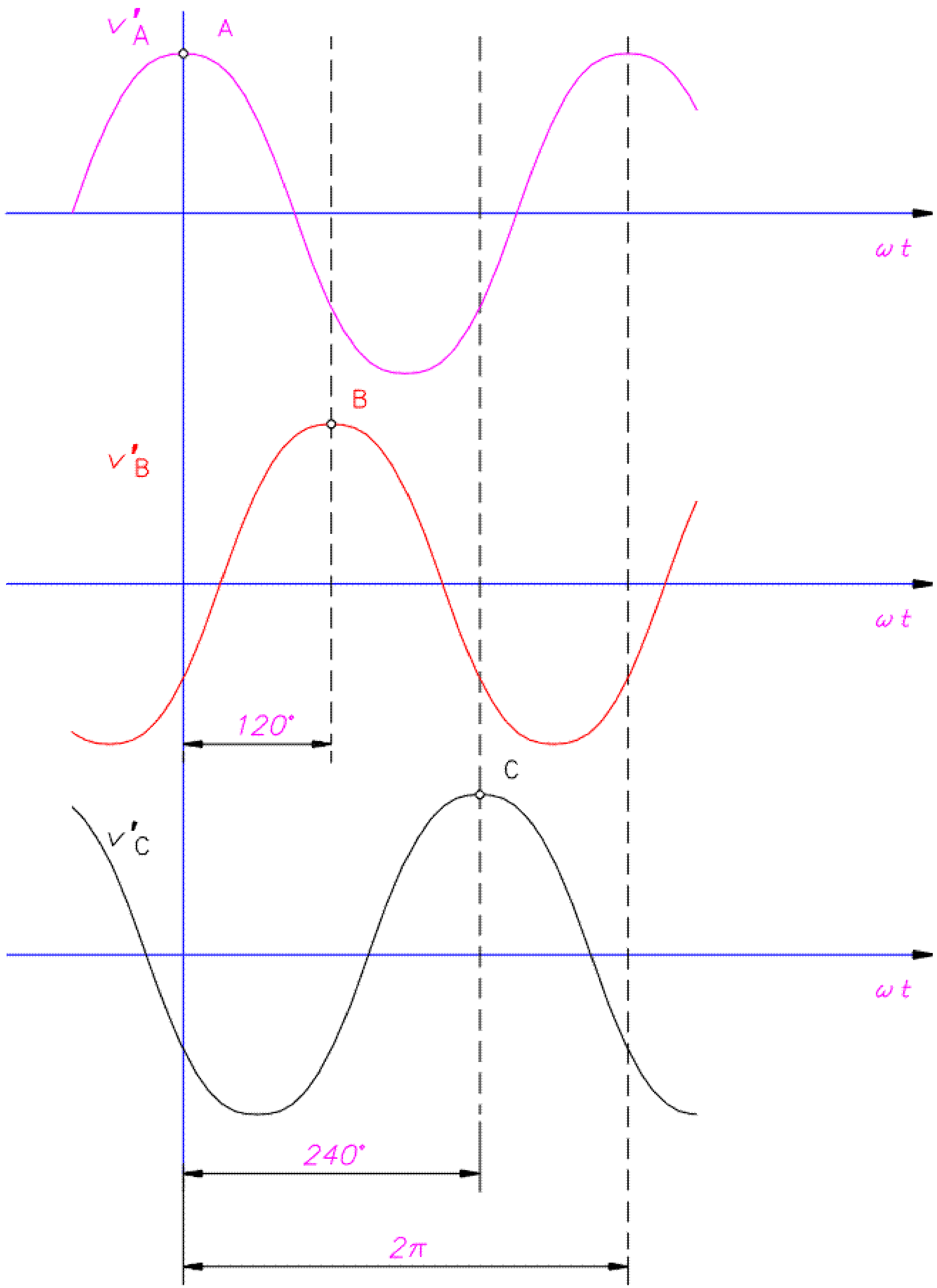
taljenje željeza feritne jezgre

VIŠEFAZNE IZMJENIČNE STRUJE

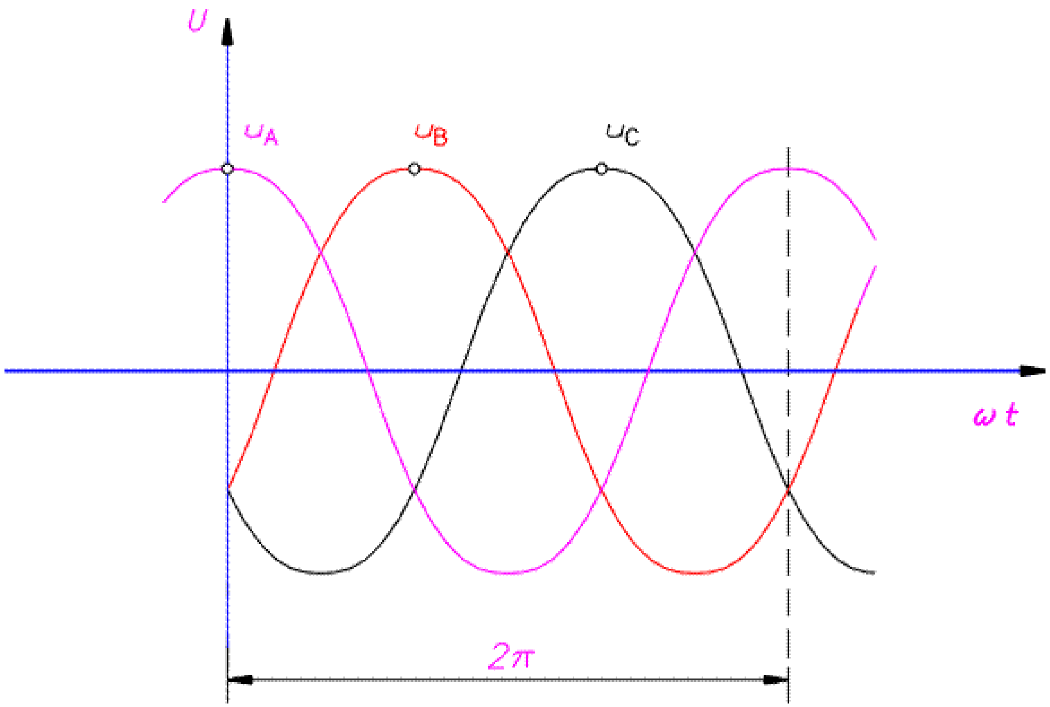




$$u = B \cdot l \cdot v' \quad (\mathrm{V})$$



zbroj trenutnih vrijednosti



$$u_A = B \cdot l \cdot v_A'$$

$$u_B = B \cdot l \cdot v_B'$$

$$u_C = B \cdot l \cdot v_C'$$

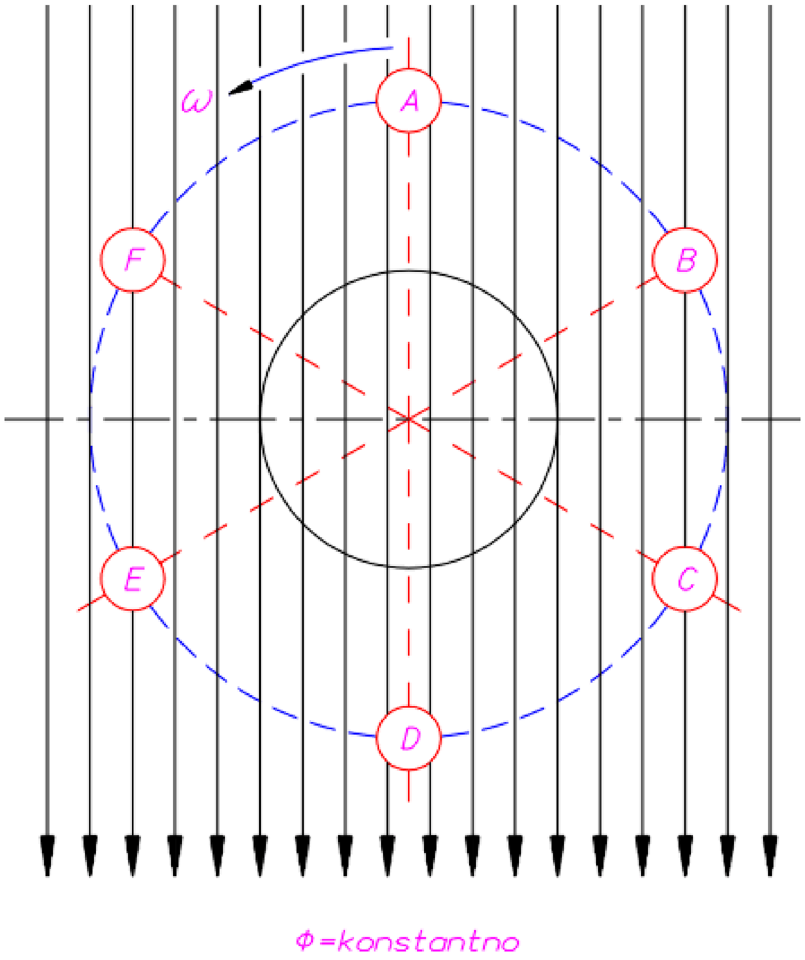
$$v_A' = v \cdot sin\frac{\pi}{2} = v$$

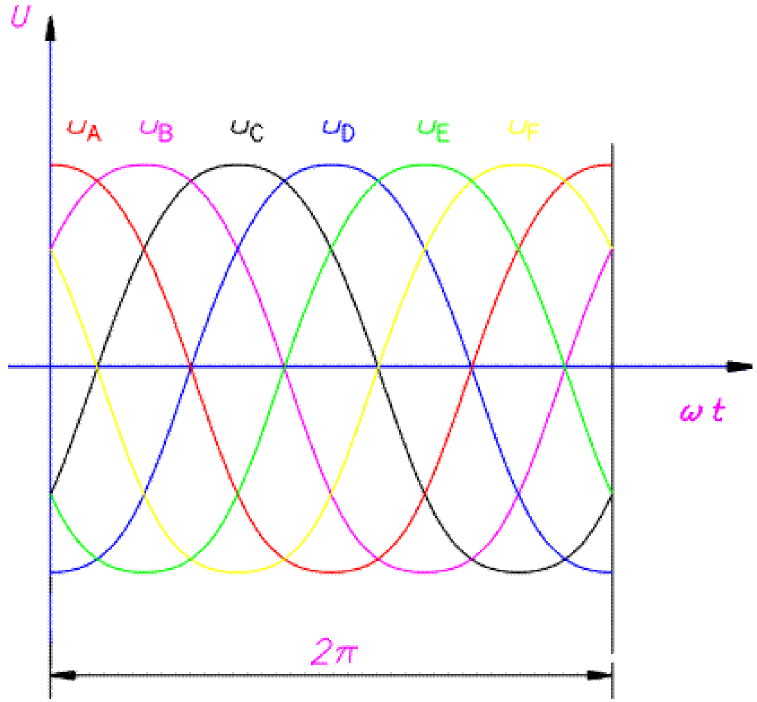
$$v_B' = v \cdot sin\left(\frac{\pi}{2} - 2\frac{\pi}{3}\right) = v \cdot sin\frac{\pi}{6}$$

$$v_C' = v \cdot sin\left(\frac{\pi}{2} - 4\frac{\pi}{3}\right) = v \cdot sin\left(-5\frac{\pi}{6}\right) = -v \cdot sin\frac{\pi}{6}$$

$$u_A + u_B + u_C = B \cdot l \cdot (v_A' + v_B' + v_C') = B \cdot l \cdot v\left(1 - sin\frac{\pi}{6} - sin\frac{\pi}{6}\right) = B \cdot l \cdot v(1 - 0.5 - 0.5) = 0$$

šesterofazni sustav



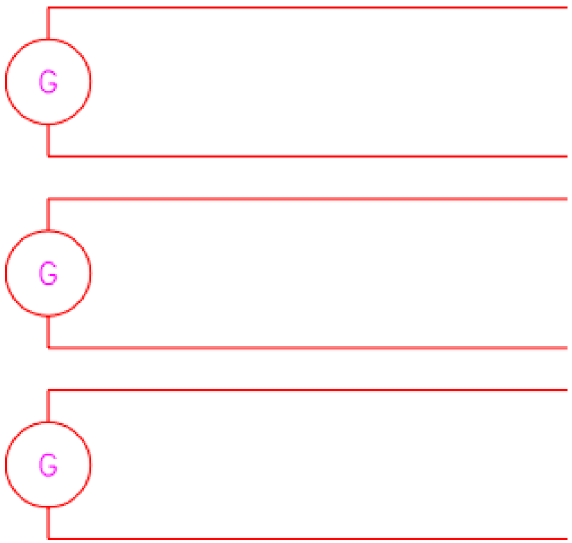


$$\underline{U}_A = -\underline{U}_D$$

$$\underline{U}_B = -\underline{U}_E$$

$$\underline{U}_C = -\underline{U}_F$$

TROFAZNI SUSTAV

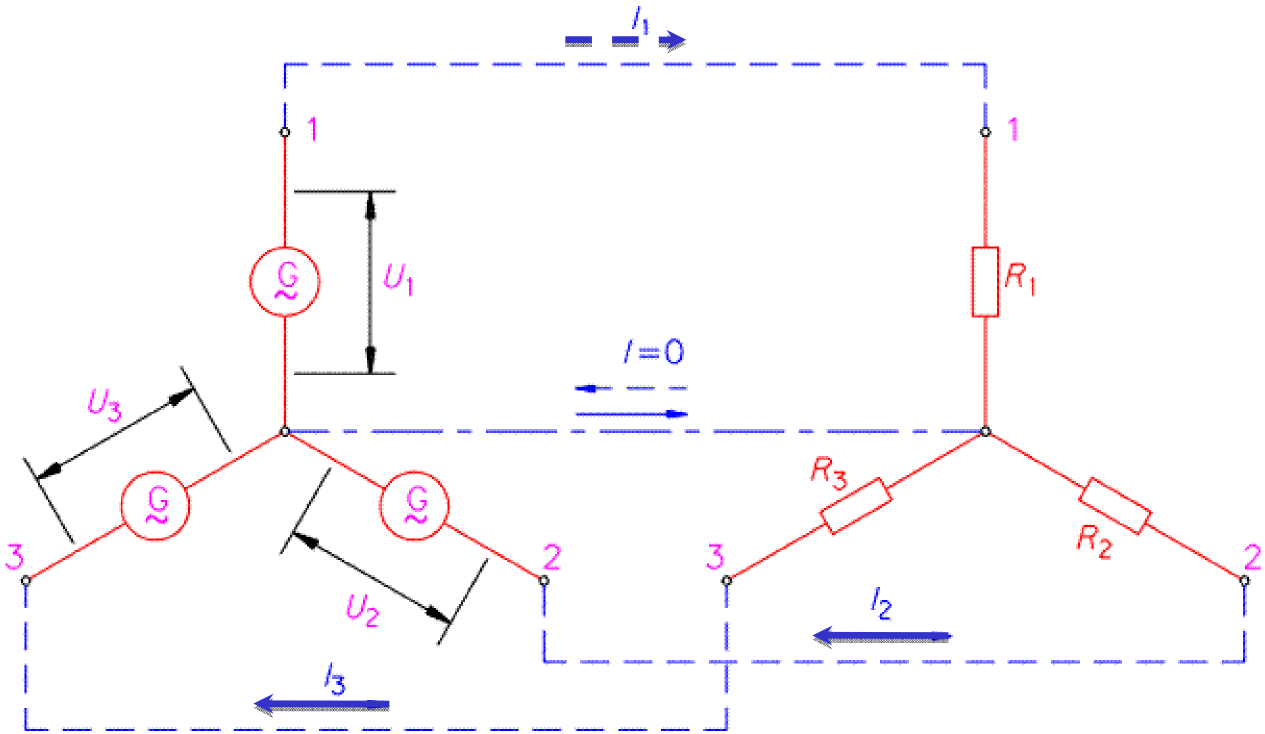


$$u_1 = \sqrt{2} \cdot U \cdot \sin \omega t$$

$$u_2 = \sqrt{2} \cdot U \cdot \sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$u_3 = \sqrt{2} \cdot U \cdot \sin\left(\omega t - \frac{4\pi}{3}\right)$$

Spoj u zvijezdu (Y)



trenutne vrijednosti

napon među fazama

$$u_1 + u_2 + u_3 = 0$$

struja kroz nul vodič

$$i = i_1 + i_2 + i_3 = 0$$

za struje kroz nul vodič

$$I_1 = \frac{U_1}{R_1}$$

$$I_2 = \frac{U_2}{R_2}$$

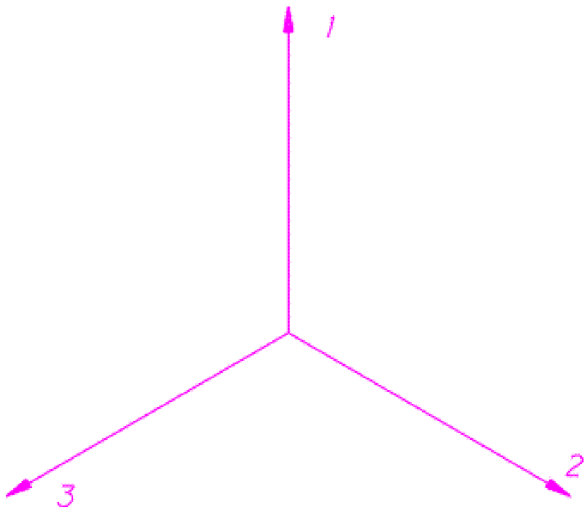
$$I_3 = \frac{U_3}{R_3}$$

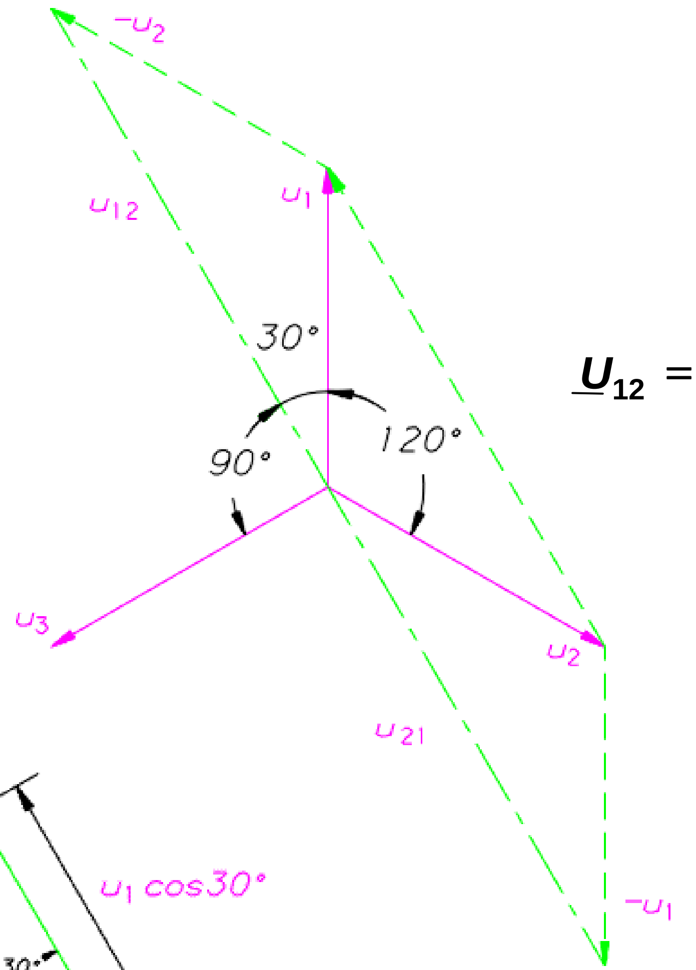
uz $R_1 = R_2 = R_3 = R_f$ i $U_1 = U_2 = U_3 = U_f$ slijedi $I_1 = I_2 = I_3$

tako da je $\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$ kod simetričnog sustava i bez nul vodiča

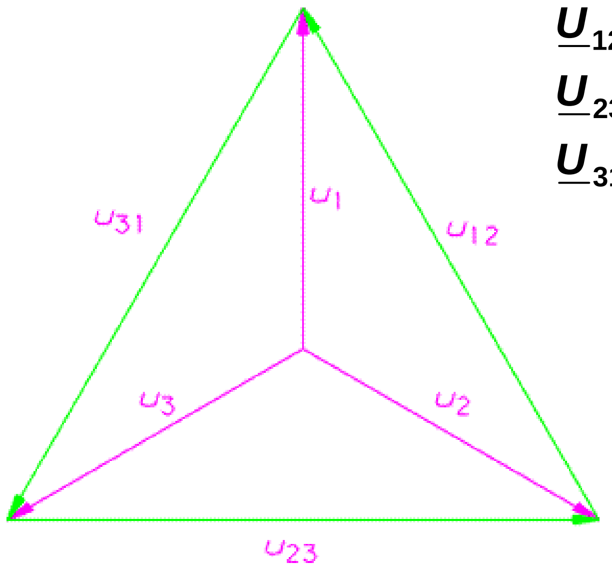
struja faze ovisi o faznom naponu i otpru $I_f = \dfrac{U_f}{R_f}$

kazalični prikaz trofaznog sustava spojenog u zvijezdu

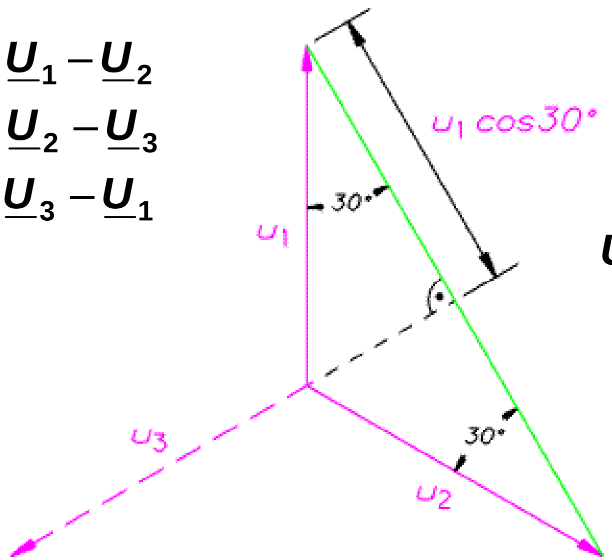




$$\underline{U}_{12} = -\underline{U}_{21}$$



$$\underline{U}_{12} = \underline{U}_1 - \underline{U}_2$$
$$\underline{U}_{23} = \underline{U}_2 - \underline{U}_3$$
$$\underline{U}_{31} = \underline{U}_3 - \underline{U}_1$$

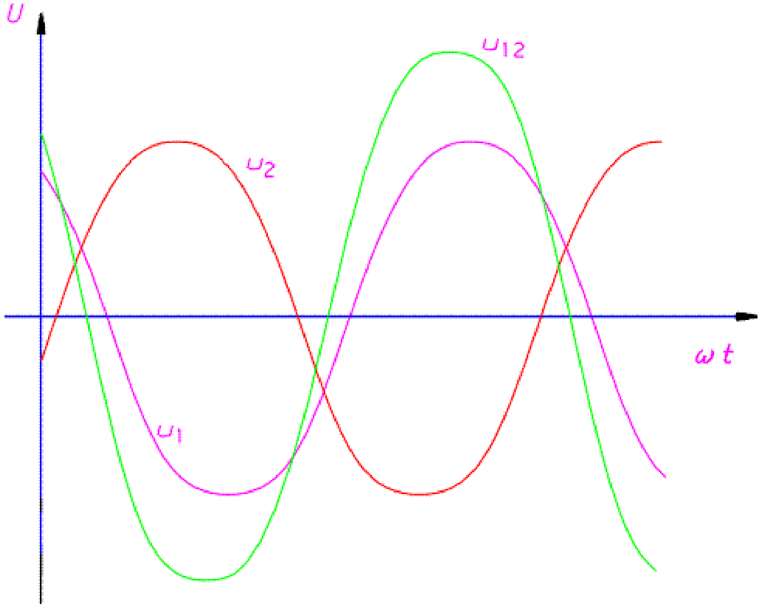


$$U_{12} = U_1 \cdot cos\,30^o + U_2 \cdot cos\,30^o$$
$$U_{12} = \sqrt{3} \cdot U_f$$
$$U = \sqrt{3} \cdot U_f$$

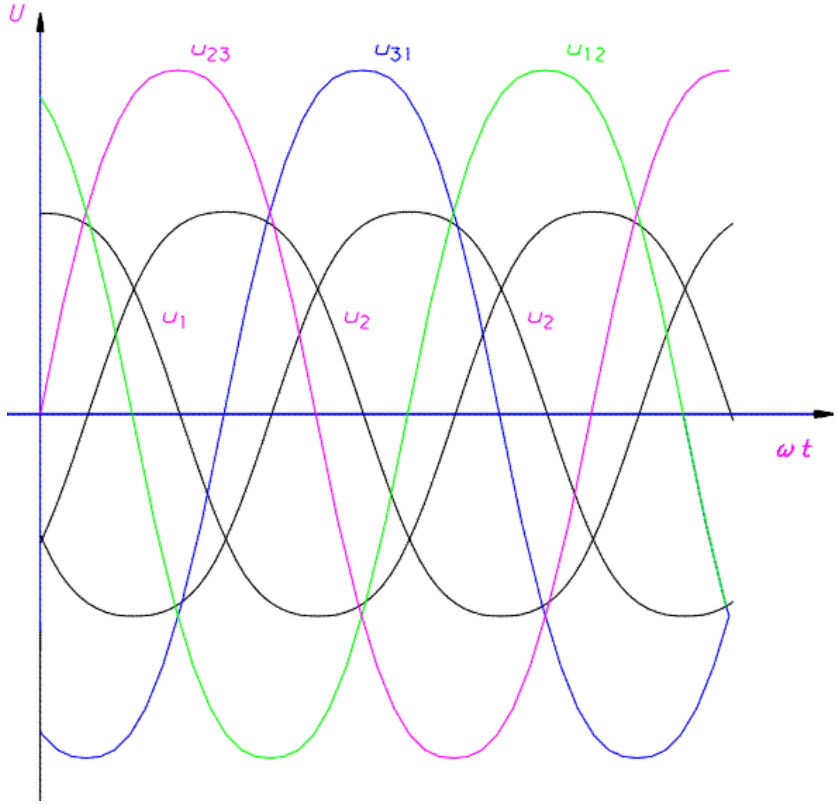
U - napon među fazama

U_f - napon pojedine faze

trenutne vrijednosti napona među fazama



jednake zakonitosti za fazne i međufazne napone ali međufazni za $\sqrt{3}$ veći

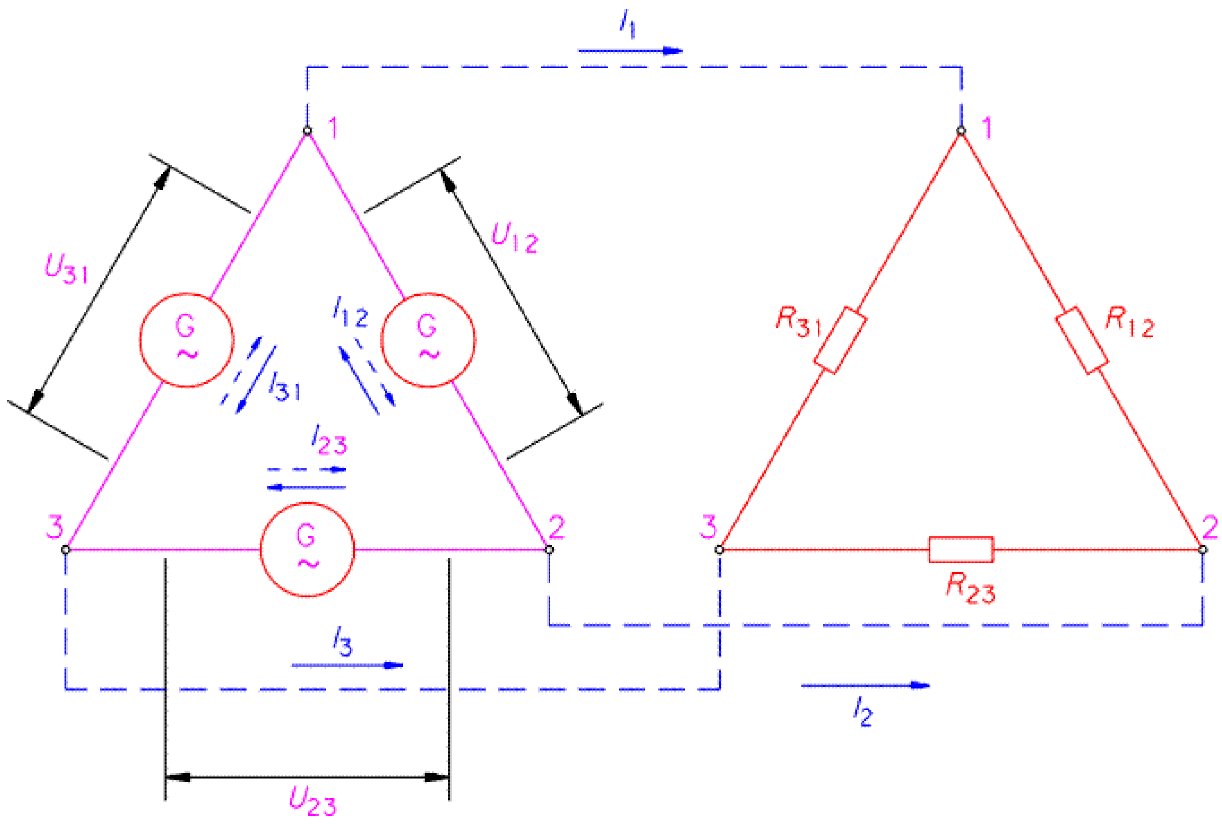


trenutne vrijednosti faznih i međufaznih napona kod trofaznog sustava

Spoj u trokut (Δ)

$$\sum \underline{U}_{(1,2,3)} = 0$$

$$\sum \underline{I}_{(1,2,3)} = 0$$

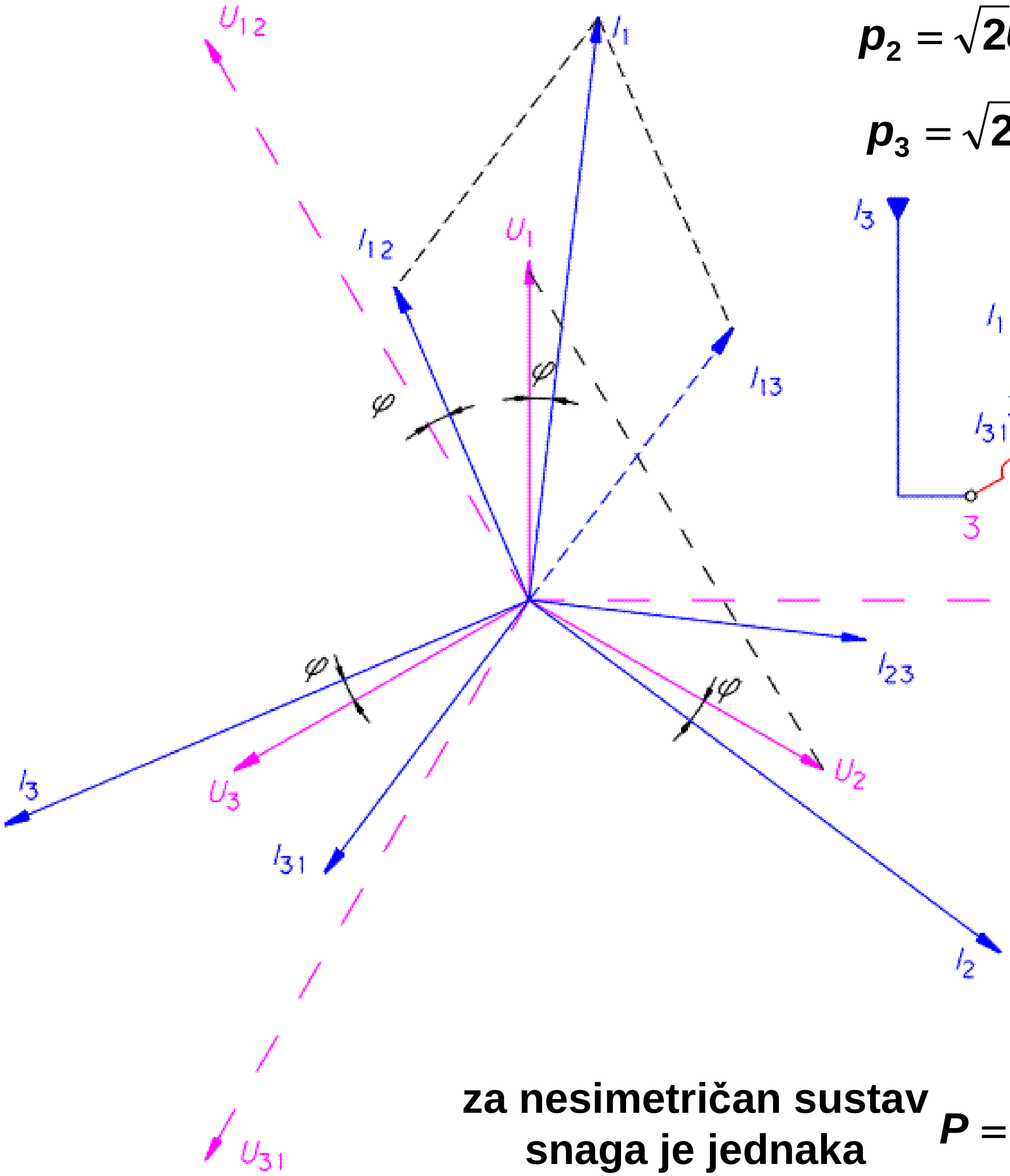


$$I_1 = I_2 = I_3 = \sqrt{3} \cdot I_{12} = \sqrt{3} \cdot I_{23} = \sqrt{3} \cdot I_{31} \qquad I = \sqrt{3} \cdot I_f$$

I - struja sustava u linijskim vodičima

I_f - struja pojedine faze izvora ili trošila

SNAGA TROFAZNOG SUSTAVA

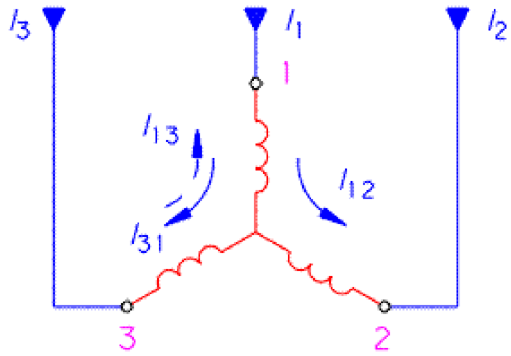


za simetričan sustav

$$p_1 = \sqrt{2}U_1 \cdot \sin \omega t \cdot \sqrt{2}I_1 \cdot \sin(\omega t - \varphi)$$

$$p_2 = \sqrt{2}U_2 \cdot \left(\sin \omega t - 120^o\right) \cdot \sqrt{2}I_2 \cdot \sin\left(\omega t - 120^o - \varphi\right)$$

$$p_3 = \sqrt{2}U_3 \cdot \sin\left(\omega t - 240^o\right) \cdot \sqrt{2}I_3 \cdot \sin\left(\omega t - 240^o - \varphi\right)$$



uz

$$U_1 = U_2 = U_3 = U_f$$

i

$$I_1 = I_2 = I_3 = I_f$$

imamo

$$P = \Sigma p = 3 \cdot U_f \cdot I_f \cdot \cos \varphi$$

za Y je $U_f = \dfrac{U}{\sqrt{3}}$

za Δ je $I_f = \dfrac{I}{\sqrt{3}}$

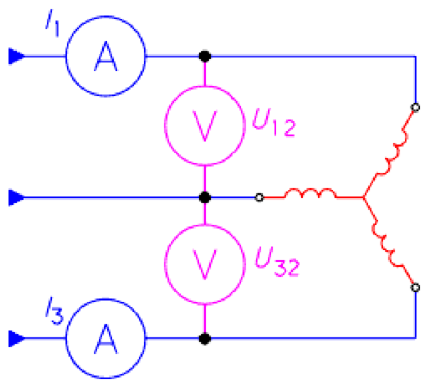
$$P = \sqrt{3} \cdot U \cdot I \cdot \cos \varphi$$

U - napon među fazama LINIJSKI NAPON

I - struja sustava LINIJSKA STRUJA

za nesimetričan sustav snaga je jednaka

$$P = U_1 \cdot I_1 \cdot \cos \varphi_1 + U_2 \cdot I_2 \cdot \cos \varphi_2 + U_3 \cdot I_3 \cdot \cos \varphi_3$$



prema slici ukupna snaga sustava

$$P = U_{12} \cdot I_1 \cdot cos(30^o - \varphi) + U_{32} \cdot I_3 \cdot cos(30^o + \varphi)$$

ako vrijedi $U_{12} = U_{32} = U$ i $I_1 = I_3 = I$

može se napisati da je

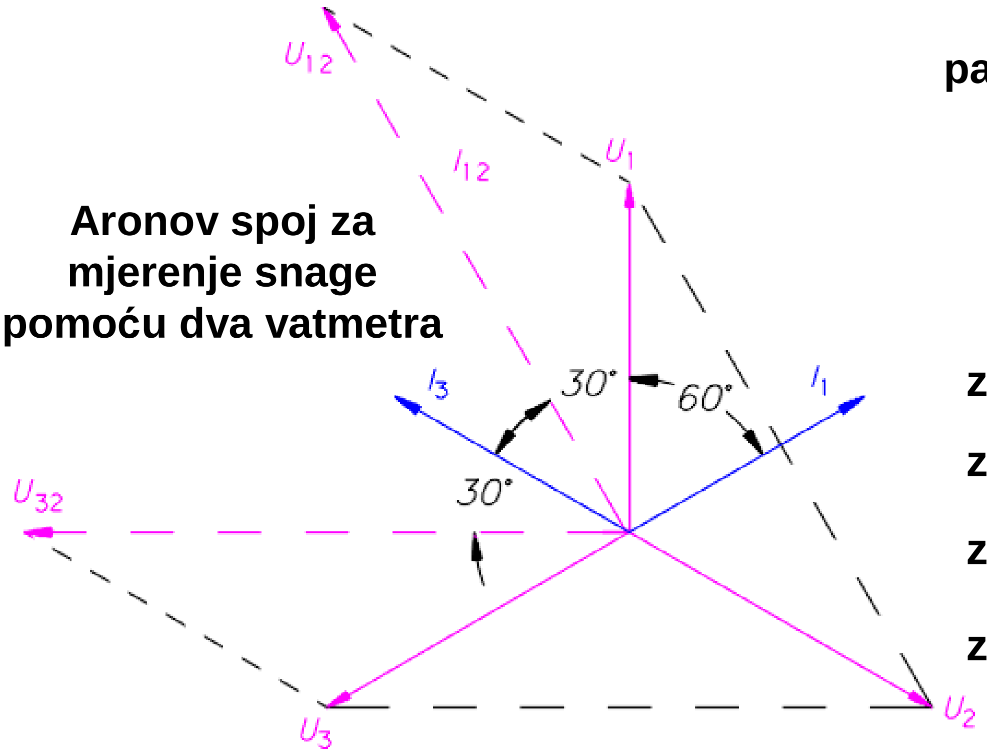
$$P = U \cdot I \cdot (cos\,30^o \cdot cos\,\varphi + sin\,30^o \cdot sin\,\varphi) + U \cdot I \cdot (cos\,30^o \cdot cos\,\varphi - sin\,30^o \cdot sin\,\varphi)$$

sređeno $P = \sqrt{3} \cdot U \cdot I \cdot cos\,\varphi$

pa vrijedi $P = \sqrt{3} \cdot U \cdot I \cdot cos\,\varphi = P_1 + P_2$

$$P_1 = \underline{U}_{12} \cdot \underline{I}_1 \cdot cos(30^o - \varphi)$$
$$P_2 = \underline{U}_{32} \cdot \underline{I}_3 \cdot cos(30^o + \varphi)$$

Aronov spoj za mjerenje snage pomoću dva vatmetra



za $\varphi = 0^o$, $cos\,\varphi = 1$ $P_1 = P_2$ $P = P_1 + P_2$

za $\varphi < 60^o$, $cos\,\varphi > 0,5$ $P = P_1 + P_2$

za $\varphi = 60^o$, $cos\,\varphi = 0,5$ $P = P_1$

za $\varphi > 60^o$, $cos\,\varphi < 0,5$ $P = P_1 - P_2$

Usporedba spojeva

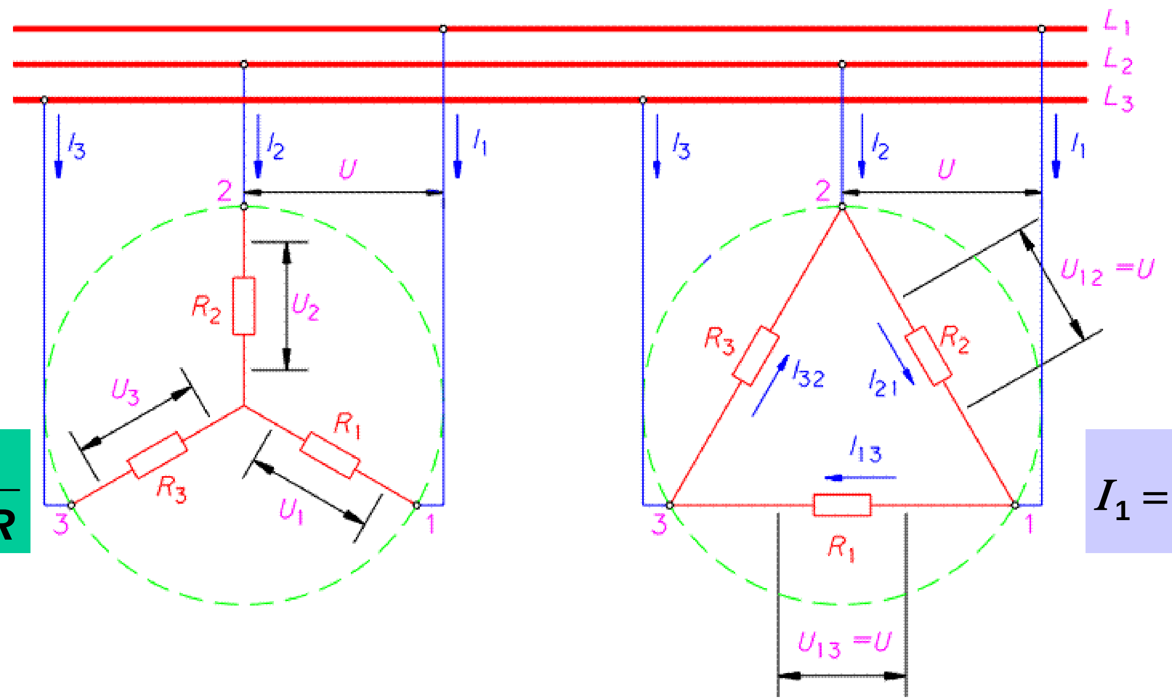
$$R = R_1 = R_2 = R_3$$

zvijezda

$$U = \sqrt{3} \cdot U_f$$

$$I = I_f$$

$$I_1 = I_2 = I_3 = \frac{U}{\sqrt{3} \cdot R}$$



trokut

$$U = U_f$$

$$I = \sqrt{3} \cdot I_f$$

$$I_1 = I_2 = I_3 = \frac{\sqrt{3} \cdot U}{R}$$

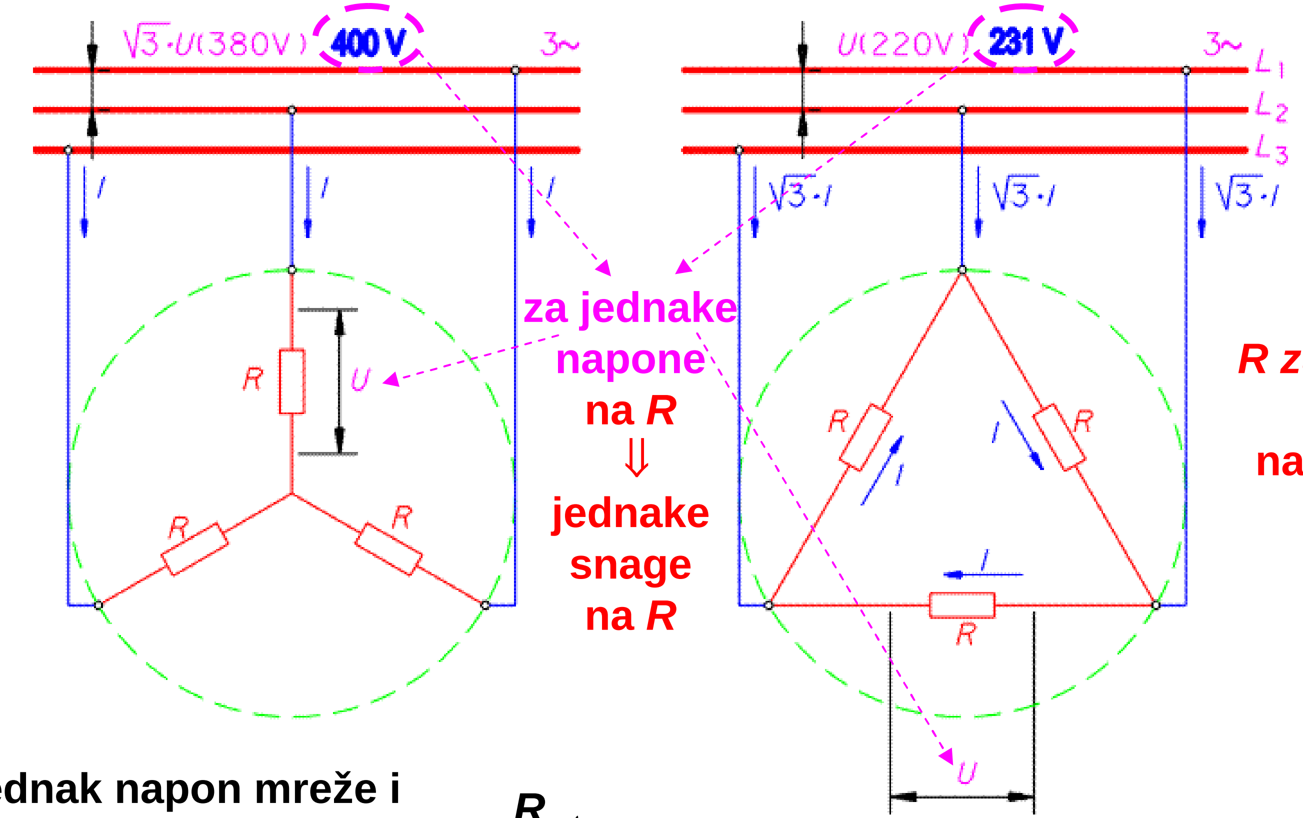
$$P_Y = \sqrt{3} \cdot U \cdot I_{1,2,3} = \sqrt{3} \cdot U \cdot \frac{U}{\sqrt{3} \cdot R} = \frac{U^2}{R}$$

$$P_\Delta = \sqrt{3} \cdot U \cdot I_{1,2,3} = \sqrt{3} \cdot U \cdot \frac{\sqrt{3} \cdot U}{R} = 3\frac{U^2}{R}$$

$$\frac{P_\Delta}{P_Y} = 3$$

otpori u spoju Δ 3 x više opterećeni nego u spoju Y

Niskonaponska mreža



***R* za 231V u Δ**
može
na 400V u Y

za jednak napon mreže i čisto radno opterećenje za jednake snage treba biti

$$\frac{R_{\Delta}}{R_{Y}} = 3$$

za kompleksno opterećenje

$$Z_{1Y} = Z_{2Y} = Z_{3Y} = \underline{Z}_{Y} = R_{Y} + jX_{Y} \qquad Z_{1\Delta} = Z_{2\Delta} = Z_{3\Delta} = \underline{Z}_{\Delta} = R_{\Delta} + jX_{\Delta}$$

Usporedba sustava za prijenos el. energije

Simboli	DC	AC 1f	AC 3f
Broj vodiča	2 vodiča	2 vodiča	3 vodiča
Snaga prijenosa	$U\,I$	$U\,I\cos\varphi$	$\sqrt{3}\,U\,I\cos\varphi$
Mogućnost proizvodnje visokog napona	- +	+	+
Faktor snage – cosφ	+	-	-
Mogućnost generiranja okretnog mag. polja	-	(+)	+
Mogućnost isključenja struje	-	+	+
Utjecaj L i C	+	-	-
Mogućnost sinkronizacije	+	-	-
Naprezanje izolacije	+	-	-